Unit 10 · Trigonometric Functions Cheat Sheet

Unit 10 focus: radians, the unit circle, reference angles and trig values of any angle

1 KEY VOCABULARY

radian	Angle whose arc equals the radius. 2π rad = 360°.	$\pi = 180^\circ$
coterminal	Same terminal side; differ by 360° (2π).	$30^\circ, 390^\circ$
reference angle	Acute angle to the x-axis.	$150^\circ \to 30^\circ$
unit circle	Radius 1: the point is $(\cos\theta, \sin\theta)$.	$(0,1)$ at 90°
amplitude / period	Half the height / length of one cycle.	3 / π

2 DEGREES ↔ RADIANS

Degrees to radians: × $\frac{\pi}{180}$. 150°	$\frac{5\pi}{6}$
Radians to degrees: × $\frac{180}{\pi}$. $\frac{7\pi}{4}$	315°
Coterminal with $-\frac{\pi}{3}$ in $[0, 2\pi)$	$\frac{5\pi}{3}$
Arc length $s = r\theta$: $r = 6$, $\theta = \frac{2\pi}{3}$	4π

3 UNIT CIRCLE (QUADRANT I)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.

Signs: All positive in QI, **S**ine in QII, **T**angent in QIII, **C**osine in QIV.

4 ANY ANGLE: REFERENCE ANGLES

$\sin 210^\circ$: ref 30°, QIII (sin < 0)	$-\frac{1}{2}$
$\cos\frac{3\pi}{4}$: ref $\frac{\pi}{4}$, QII (cos < 0)	$-\frac{\sqrt{2}}{2}$
$\tan\frac{5\pi}{3}$: ref $\frac{\pi}{3}$, QIV (tan < 0)	$-\sqrt{3}$

5 POINT ON THE TERMINAL SIDE

For (x, y): $r = \sqrt{x^2 + y^2}$, $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$.

Example: The terminal side passes through $(-3, 4)$.
r: $\sqrt{9 + 16} = 5$

Answer: $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Set the calculator to the right mode (degrees or radians).
- Reference angles are always measured to the x-axis.
- $\sin^2\theta$ means $(\sin\theta)^2$.

✘ Common mistakes

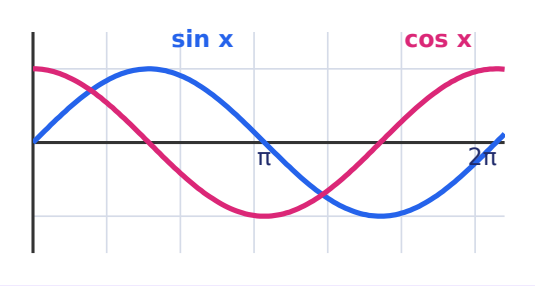
- $\sin^{-1} x = \frac{1}{\sin x}$: it is the inverse (arcsin), not the reciprocal
- $\sin 2x = 2\sin x$: not true in general
- Period of $\sin 2x$ is 4π: it is $\frac{2\pi}{2} = \pi$

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More practice: sine and cosine graphs, amplitude, period, phase shift, identities, equations, inverses and modeling

1 GRAPHS OF SINE & COSINE

Both have amplitude 1 and period 2π. $\sin$ starts at 0 going up; $\cos$ starts at its maximum. Range $[-1, 1]$.



2 AMPLITUDE, PERIOD, SHIFTS

$y = a\sin\big(b(x-h)\big) + k$: amplitude $|a|$, period $\frac{2\pi}{b}$, phase shift h, midline $y = k$.

Example: $y = 3\sin\big(2(x - \frac{\pi}{4})\big) + 1$
Read: amplitude 3, period π, right $\frac{\pi}{4}$, midline $y = 1$
Answer: max 4, min -2

3 IDENTITIES

$\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Use the quadrant for the sign.

Example: $\sin\theta = \frac{3}{5}$, θ in QII
$\cos^2\theta$: $1 - \frac{9}{25} = \frac{16}{25}$, cos < 0 in QII
Answer: $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

4 SOLVING TRIG EQUATIONS

$2\sin x - 1 = 0$ on $[0, 2\pi)$	$x = \frac{\pi}{6},\ \frac{5\pi}{6}$
$2\cos^2 x - \cos x - 1 = 0$: $(2\cos x + 1)(\cos x - 1) = 0$	$x = 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}$
All solutions of $\tan x = 1$	$x = \frac{\pi}{4} + n\pi$

5 INVERSE TRIG FUNCTIONS

Function	Range	Example
$\sin^{-1} x$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$
$\cos^{-1} x$	$[0, \pi]$	$\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$
$\tan^{-1} x$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$\tan^{-1}(-1) = -\frac{\pi}{4}$

6 MODELING: FERRIS WHEEL

Midline = center height, amplitude = radius, period = time for one turn. Starting at the bottom: use $-\cos$.

Example: Diameter 50 m, center 30 m high, one turn every 10 min, start at the bottom.
Model: $h(t) = -25\cos\left(\frac{\pi}{5}t\right) + 30$
Check: $h(0) = 5$, $h(5) = 55$
Answer: $h(2.5) = 30$ **m (level with the center)**