Unit 3 · Anticipating Patterns Cheat Sheet

Unit 3 focus: probability rules, conditional probability, independence and simulation

1 KEY VOCABULARY

probability	Long-run relative frequency of an outcome.	P(heads) = 0.5
law of large numbers	With many trials, the proportion settles near the true probability.	10,000 flips ≈ 50% heads
sample space	The list of all possible outcomes.	{1, 2, 3, 4, 5, 6}
mutually exclusive	Events that cannot happen together.	roll a 2 and a 5
independent	Knowing one event doesn't change the other's probability.	$P(A \mid B) = P(A)$

2 PROBABILITY RULES

Rule	Formula
Complement	$P(A^C) = 1 - P(A)$
Addition (or)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Multiplication (and)	$P(A \cap B) = P(A) \cdot P(B \mid A)$
Conditional	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

"At least one": $1 - P(\text{none})$. Three free throws at 80%: $1 - 0.2^3 = 0.992$.

3 CONDITIONAL PROBABILITY FROM A TABLE

	Pass	Fail	Total
Studied	42	8	50
Didn't study	18	12	30
Total	60	20	80

$P(\text{pass} \mid \text{studied}) = \frac{42}{50} = 0.84$, but $P(\text{studied} \mid \text{pass}) = \frac{42}{60} = 0.70$. The "given" group is the denominator.

4 CHECKING INDEPENDENCE

A and B are independent if $P(A \mid B) = P(A)$, or $P(A \cap B) = P(A) \cdot P(B)$.

Example: Using the table: is passing independent of studying?
$P(\text{pass})$: $\frac{60}{80} = 0.75$
$P(\text{pass} \mid \text{studied})$: 0.84
Compare: $0.84 \neq 0.75$
Answer: not independent

5 SIMULATION

- **State** the question about a chance process.
- **Plan**: assign digits to outcomes. For $p = 0.3$: digits 0–2 = success, 3–9 = failure. Say what one trial is and what you record.
- **Do** many trials (repeats allowed when trials are independent).
- **Conclude**: estimated probability = proportion of trials with the event.

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Draw a tree diagram for multi-stage problems.
- Probabilities are between 0 and 1.
- Mutually exclusive events (with nonzero P) are never independent.

✗ Common mistakes

- $P(A \mid B) = P(B \mid A)$: different denominators
- Adding for "and": multiply for "and"
- Forgetting the overlap in "or": subtract $P(A \cap B)$

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More practice: random variables, binomial and geometric models, and sampling distributions

1 DISCRETE RANDOM VARIABLES

$\mu_X = \sum x\,p$, $\sigma_X = \sqrt{\sum (x-\mu)^2 p}$. The probabilities must sum to 1.

Example: X = 0, 1, 2, 3 with p = 0.1, 0.3, 0.4, 0.2
Mean: $0 + 0.3 + 0.8 + 0.6 = 1.7$
Variance: $0.289 + 0.147 + 0.036 + 0.338 = 0.81$
Answer: $\mu_X = 1.7$, $\sigma_X = 0.9$

2 TRANSFORMING AND COMBINING

Random variable	Mean and SD
$Y = a + bX$	$\mu_Y = a + b\mu_X$, $\sigma_Y = \lvert b\rvert \sigma_X$
$X + Y$ or $X - Y$	means add or subtract: $\mu_X \pm \mu_Y$
$X \pm Y$, independent	variances **always add**: $\sigma_X^2 + \sigma_Y^2$

$\sigma_X = 3$, $\sigma_Y = 4$: $\sigma_{X-Y} = \sqrt{9+16} = 5$ (not $4-3$).

3 BINOMIAL DISTRIBUTION

BINS: Binary, Independent, fixed Number, Same p.
$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$, $\mu = np$, $\sigma = \sqrt{np(1-p)}$.

Example: 5 free throws at $p = 0.7$. $P(X=4)$?
Formula: $\binom{5}{4}(0.7)^4(0.3)^1$
Mean, SD: $5(0.7) = 3.5$, $\sqrt{1.05} \approx 1.02$
Answer: $P(X=4) \approx 0.360$

4 GEOMETRIC DISTRIBUTION

Count trials until the first success: $P(X=k) = (1-p)^{k-1}p$, $\mu = \frac{1}{p}$.

Example: 20% of cereal boxes hold a prize. First prize in box 3?
Formula: $(0.8)^2(0.2)$
Mean: $\frac{1}{0.2} = 5$ boxes
Answer: 0.128

5 SAMPLING DISTRIBUTIONS

Statistic	Center, spread, shape
$\hat{p}$	$\mu = p$, $\sigma = \sqrt{\frac{p(1-p)}{n}}$; Normal if $np \geq 10$ and $n(1-p) \geq 10$
$\bar{x}$	$\mu = \mu$, $\sigma = \frac{\sigma}{\sqrt{n}}$; Normal if population is Normal or $n \geq 30$ (CLT)

10% condition: $n \leq 10\%$ of the population. Larger $n \Longrightarrow$ less variability; an **unbiased** statistic is centered at the parameter.

6 CENTRAL LIMIT THEOREM

For large n, $\bar{x}$ is approximately Normal even if the population is skewed.

Example: $\mu = 40$, $\sigma = 12$, $n = 36$. $P(\bar{x} > 44)$?
SD of $\bar{x}$: $\frac{12}{\sqrt{36}} = 2$
z: $\frac{44-40}{2} = 2$
Answer: $P(Z > 2) \approx 0.0228$

