Unit 8 · Sequences & Series Cheat Sheet

Unit 8 focus: arithmetic and geometric sequences, recursive and explicit formulas

1 KEY VOCABULARY

sequence	An ordered list of terms $a_1, a_2, a_3, \ldots$.	$2, 5, 8, \ldots$
common difference ⟨d⟩	Constant added each step (arithmetic).	$d = 3$
common ratio ⟨r⟩	Constant multiplier each step (geometric).	$r = 2$
series	The sum of the terms of a sequence.	$2 + 5 + 8$
recursive formula	Gives a term from the one before it.	$a_n = a_{n-1} + 3$

2 ARITHMETIC SEQUENCES

$a_n = a_1 + (n-1)d$. Linear: equal differences.

Example: $7, 11, 15, \ldots$: find a_{20}.
$d = 4$: $a_{20} = 7 + 19(4)$

Answer: $a_{20} = 83$

3 GEOMETRIC SEQUENCES

$a_n = a_1 \cdot r^{n-1}$. Exponential: equal ratios.

Example: $3, 6, 12, \ldots$: find a_{10}.
$r = 2$: $a_{10} = 3 \cdot 2^9$

Answer: $a_{10} = 1536$

4 RECURSIVE ↔ EXPLICIT

Recursive	Explicit
$a_1 = 5,\ a_n = a_{n-1} - 3$	$a_n = 8 - 3n$
$a_1 = 2,\ a_n = 3a_{n-1}$	$a_n = 2 \cdot 3^{n-1}$
$a_1 = 64,\ a_n = \frac{1}{2}a_{n-1}$	$a_n = 64\left(\frac{1}{2}\right)^{n-1}$

Check: a_3 is -1, 18 and 16 from either form.

5 WHICH KIND?

$2, 5, 8, 11, \ldots$ (differences 3)	**arithmetic,** $d = 3$
$81, 27, 9, 3, \ldots$ (ratios $\frac{1}{3}$)	**geometric,** $r = \frac{1}{3}$
$1, 4, 9, 16, \ldots$	**neither**
$5, -10, 20, -40, \ldots$	**geometric,** $r = -2$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- n counts terms: the first term is $n = 1$.
- Find r as later ÷ earlier.
- A recursive formula needs a starting term.

✘ Common mistakes

- $a_n = a_1 + nd$: it is $(n-1)d$
- $a_n = (a_1 r)^{n-1}$: only r is raised to the power
- $r = 3$ for $81, 27, 9$: $r = \frac{27}{81} = \frac{1}{3}$

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More practice: arithmetic and geometric series, sigma notation, infinite series and applications

1 ARITHMETIC SERIES

$S_n = \frac{n(a_1 + a_n)}{2}$: number of terms × average of first and last.

Example: $3 + 7 + 11 + \cdots + 79$
Count: $n = \frac{79-3}{4} + 1 = 20$
Sum: $\frac{20(3+79)}{2}$

Answer: 820

2 GEOMETRIC SERIES

$S_n = \frac{a_1(1 - r^n)}{1 - r}$, $r \neq 1$.

Example: First 8 terms of $2 + 6 + 18 + \ldots$
$r = 3$: $S_8 = \frac{2(1 - 3^8)}{1 - 3}$

Answer: 6560

3 SIGMA NOTATION

$\sum_{k=1}^{5}(2k + 1) = 3 + 5 + 7 + 9 + 11$	35
$\sum_{k=1}^{4} 3 \cdot 2^{k-1} = 3 + 6 + 12 + 24$	45
$\sum_{k=1}^{100} k$	5050

4 INFINITE GEOMETRIC SERIES

Converges only when $|r| < 1$: $S = \frac{a_1}{1 - r}$. If $|r| \geq 1$ it diverges (no sum).

Example: $12 + 6 + 3 + \ldots$ and $0.\overline{3} = 0.3 + 0.03 + \ldots$
$r = \frac{1}{2}$: $\frac{12}{1-1/2} = 24$
$r = \frac{1}{10}$: $\frac{0.3}{0.9} = \frac{1}{3}$

Answer: 24 **and** $\frac{1}{3}$

5 APPLICATION: SEATING

Equal increases each row: arithmetic. Total seats: an arithmetic series.

Example: Row 1 has 20 seats; each row has 2 more; 25 rows.
Last row: $a_{25} = 20 + 24(2) = 68$
Total: $\frac{25(20+68)}{2}$

Answer: 1100 **seats**

6 APPLICATION: BOUNCING BALL

Each bounce is a fixed percent of the last: geometric. Count up and down trips.

Example: Dropped from 10 ft; each rebound is 60% as high.
Rebounds: $6 + 3.6 + \cdots = \frac{6}{1-0.6} = 15$
Total: $10 + 2(15)$ (up and down)

Answer: 40 **ft**