Unit 7 · Rational Functions Cheat Sheet

Unit 7 focus: simplifying, multiplying, dividing, adding and subtracting rational expressions, and solving rational equations

1 KEY VOCABULARY

rational expression	A ratio of polynomials $\frac{P}{Q}$, $Q \neq 0$.	$\frac{x+1}{x-3}$
excluded value	Makes the original denominator 0.	$x = 3$
vertical asymptote	Line $x = a$ the graph approaches but never crosses.	$x = 3$
horizontal asymptote	Value y approaches as $x \to \pm\infty$.	$y = 1$
hole	A missing point from a canceled factor.	$(2, 4)$

2 SIMPLIFYING

Factor completely, list excluded values **before** canceling, then cancel common factors.

Example: $\dfrac{x^2-9}{x^2+x-6}$
Factor: $\dfrac{(x-3)(x+3)}{(x+3)(x-2)}$
Exclude: $x \neq -3,\ 2$
Answer: $\dfrac{x-3}{x-2}$

3 MULTIPLYING & DIVIDING

$\dfrac{x^2-4}{x+1} \cdot \dfrac{x+1}{x-2}$	$x+2$ **(**$x \neq -1, 2$**)**
$\dfrac{x}{x-3} \div \dfrac{x^2}{x^2-9}$	$\dfrac{x+3}{x}$ **(**$x \neq 0, \pm 3$**)**
$\dfrac{2x}{5} \cdot \dfrac{15}{x^2}$	$\dfrac{6}{x}$

4 ADDING & SUBTRACTING

Find the LCD, rewrite each fraction over it, combine numerators, simplify.

Example: $\dfrac{2}{x-1} + \dfrac{3}{x+2}$
LCD: $(x-1)(x+2)$
Numerator: $2(x+2) + 3(x-1) = 5x+1$
Answer: $\dfrac{5x+1}{(x-1)(x+2)}$

5 SOLVING RATIONAL EQUATIONS

Multiply every term by the LCD, solve, and reject any excluded value.

Example: $\dfrac{x}{x-2} + 1 = \dfrac{2}{x-2}$
Times $(x-2)$**:** $x + (x-2) = 2 \to x = 2$
Check: $x = 2$ makes a denominator 0 ✘
Answer: no solution

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- State excluded values from the **original** denominators.
- Cancel factors, never terms.
- Check solutions against the excluded values.

✘ Common mistakes

- $\frac{x+5}{x} = 5$: you cannot cancel the x terms
- $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$: it is $\frac{a+b}{ab}$
- Subtracting $\frac{x-1}{x+2}$ without parentheses: distribute the minus to the whole numerator

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More practice: asymptotes and holes, graphing, transformations of 1/x, variation and work problems

1 ASYMPTOTES & HOLES

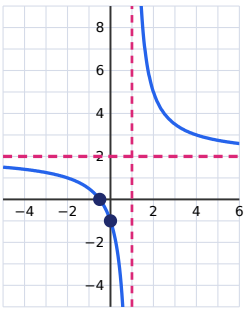
Hole	factor that cancels; plug x into the simplified form
Vertical asymptote	zero of the denominator after canceling
HA: deg top < deg bottom	$y = 0$
HA: degrees equal	$y = \frac{\text{lead coeff. top}}{\text{lead coeff. bottom}}$
deg top > deg bottom	no HA (slant if one more)

2 GRAPHING EXAMPLE

Find the asymptotes and intercepts, then test points on each side.

Example: $f(x) = \dfrac{2x+1}{x-1}$
Asymptotes: $x = 1$, $y = 2$
Intercepts: $(-\frac{1}{2}, 0)$, $(0, -1)$

Answer: two branches around $(1, 2)$



3 HOLES VS. ASYMPTOTES

A canceled factor gives a hole; a factor left in the denominator gives an asymptote.

Example: $g(x) = \dfrac{x^2 + x - 2}{x^2 - 4}$
Factor: $\dfrac{(x+2)(x-1)}{(x+2)(x-2)} = \dfrac{x-1}{x-2}$
Hole: $x = -2$: $\frac{-3}{-4} = \frac{3}{4}$

Answer: hole $(-2, \frac{3}{4})$**; VA** $x = 2$**; HA** $y = 1$

4 TRANSFORMATIONS OF 1/X

$y = \dfrac{a}{x-h} + k$ has VA $x = h$ and HA $y = k$. $a < 0$ flips the branches into the other quadrants.

Example: $y = \dfrac{3}{x+2} - 1$
Shift: left 2, down 1
Points: $(-1, 2)$, $(-3, -4)$

Answer: VA $x = -2$**, HA** $y = -1$

5 VARIATION

Inverse: $y = \frac{k}{x}$; $y = 6$ when $x = 4$. Find y when $x = 8$.	$k = 24$**,** $y = 3$
Joint: $z = kxy$; $z = 60$ when $x = 3, y = 4$.	$k = 5$
Combined: $y = \frac{kx}{z}$; $y = 10$ when $x = 5, z = 2$.	$k = 4$

6 WORK & RATE PROBLEMS

Add the rates (jobs per hour): $\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$.

Example: Pipe A fills a tank in 3 h, pipe B in 6 h. Together?
Rates: $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ tank per hour
Time: $\frac{1}{t} = \frac{1}{2}$

Answer: $t = 2$ **hours**