Unit 5 · Radicals & Rational Exponents Cheat Sheet

Unit 5 focus: simplifying radicals, radical operations, rational exponents and converting between forms

1 KEY VOCABULARY

index	The small n in $\sqrt[n]{a}$ (2 if not written).	$\sqrt[3]{8}$: 3
radicand	The expression under the radical sign.	$\sqrt{5x}$: $5x$
rational exponent	$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.	$8^{2/3} = 4$
like radicals	Same index and same radicand.	$2\sqrt{3},\ 7\sqrt{3}$
extraneous solution	Solves a step but not the original equation.	check it!

2 SIMPLIFYING RADICALS

$\sqrt{72} = \sqrt{36 \cdot 2}$	$6\sqrt{2}$
$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$	$3\sqrt[3]{2}$
$\sqrt{50x^3}\ (x \geq 0)$	$5x\sqrt{2x}$
$\sqrt[4]{32} = \sqrt[4]{16 \cdot 2}$	$2\sqrt[4]{2}$

3 RADICAL OPERATIONS

Add: $3\sqrt{12} + \sqrt{27} = 6\sqrt{3} + 3\sqrt{3}$	$9\sqrt{3}$
Multiply: $(2+\sqrt{3})(2-\sqrt{3})$	1
Rationalize: $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$	$2\sqrt{3}$
Conjugate: $\frac{4}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{4(3+\sqrt{5})}{4}$	$3+\sqrt{5}$

4 RATIONAL EXPONENTS

Expression	Meaning	Value
$27^{1/3}$	$\sqrt[3]{27}$	3
$8^{2/3}$	$(\sqrt[3]{8})^2$	4
$16^{-3/4}$	$\dfrac{1}{(\sqrt[4]{16})^3}$	$\frac{1}{8}$
$(-32)^{3/5}$	$(\sqrt[5]{-32})^3$	-8

Take the root first: the numbers stay small.

5 CONVERTING & EXPONENT RULES

$\sqrt[3]{x^2}$ as a power	$x^{2/3}$
$x^{5/4}$ as a radical	$\sqrt[4]{x^5} = x\sqrt[4]{x}$
$x^{1/2} \cdot x^{1/3}$	$x^{5/6}$
$(x^{2/3})^6$	x^4
$\dfrac{x^{3/4}}{x^{1/4}}$	$x^{1/2} = \sqrt{x}$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- $\sqrt{x^2} = |x|$; even roots of negatives are not real.
- Odd roots of negatives are real: $\sqrt[3]{-8} = -2$.
- Simplify before adding radicals.

✘ Common mistakes

- $\sqrt{9+16} = 3+4$: it is $\sqrt{25} = 5$
- $2\sqrt{3} + 3\sqrt{2} = 5\sqrt{5}$: unlike radicals do not combine
- $8^{-1/3} = -2$: it is $\frac{1}{2}$

Unit 5 · Radicals & Rational Exponents Cheat Sheet

More practice: radical equations, extraneous solutions, and square-root and cube-root graphs

1 SOLVING RADICAL EQUATIONS

Isolate the radical, raise both sides to the index, solve, then **check every answer** in the original.

Example: $\sqrt{2x+3} = x$
Square: $2x + 3 = x^2 \rightarrow (x-3)(x+1) = 0$
Check -1: $\sqrt{1} = 1 \neq -1$ ✘

Answer: $x = 3$

2 EXTRANEOUS SOLUTIONS

Squaring can create solutions that do not work. Two radicals: isolate one, square, isolate the other, square again.

Example: $\sqrt{x+7} - 1 = x$
Square: $x + 7 = (x+1)^2 \rightarrow x^2 + x - 6 = 0$
Check: $x = 2$ ✔, $x = -3$: $\sqrt{4} - 1 = 1 \neq -3$ ✘

Answer: $x = 2$

3 RATIONAL-EXPONENT EQUATIONS

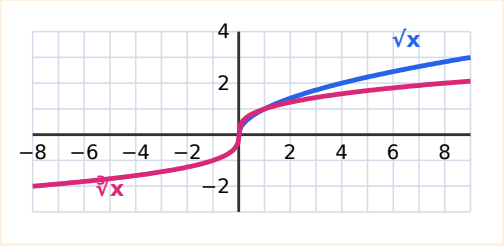
Isolate the power, then raise both sides to the reciprocal exponent. An even root gives $\pm$.

Example: $(x+1)^{2/3} = 4$
Raise to $\frac{3}{2}$: $x + 1 = \pm 4^{3/2} = \pm 8$
Check: $\sqrt[3]{8}^2 = 4$, $\sqrt[3]{-8}^2 = 4$ ✔

Answer: $x = 7$ **or** $x = -9$

4 SQUARE- & CUBE-ROOT GRAPHS

Parent	Domain	Range	Key points
$y = \sqrt{x}$	$x \geq 0$	$y \geq 0$	$(0,0), (1,1), (4,2)$
$y = \sqrt[3]{x}$	all reals	all reals	$(-8,-2), (0,0), (8,2)$

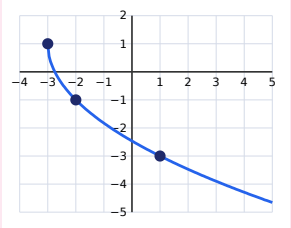


5 TRANSFORMATIONS

$y = a\sqrt{x-h} + k$ starts at (h, k). $a < 0$ reflects over the x-axis; $|a| > 1$ stretches.

Example: $y = -2\sqrt{x+3} + 1$
Start: $(-3, 1)$; next point $(-2, -1)$
Domain / range: $x \geq -3$, $y \leq 1$

Answer: left 3, up 1, reflected, stretched ×2



6 APPLICATION: PENDULUM

Period (seconds) of a pendulum L feet long: $T = 2\pi\sqrt{\frac{L}{32}}$.

Example: Find T for $L = 8$. What L gives $T = 2\pi$?
$L = 8$**:** $T = 2\pi\sqrt{\frac{1}{4}} = \pi$
$T = 2\pi$**:** $\sqrt{\frac{L}{32}} = 1 \rightarrow L = 32$

Answer: $T \approx 3.14$ **s;** $L = 32$ **ft**