Unit 11 · Quadratic Functions Cheat Sheet

Unit 11 focus: graphing quadratic functions and their key features

1 KEY VOCABULARY

parabola	The U-shaped graph of a quadratic.	$y = x^2$
vertex	The turning point (maximum or minimum).	$(2, -9)$
axis of symmetry	Vertical line through the vertex.	$x = 2$
zeros	Where $y = 0$: the x-intercepts.	$x = -1,\ 5$
discriminant	$b^2 - 4ac$: how many real solutions.	$64 > 0$

2 THREE FORMS

Form	Equation	Shows
Standard	$y = ax^2 + bx + c$	y-intercept $(0, c)$
Vertex	$y = a(x - h)^2 + k$	vertex (h, k)
Factored	$y = a(x - p)(x - q)$	zeros p and q

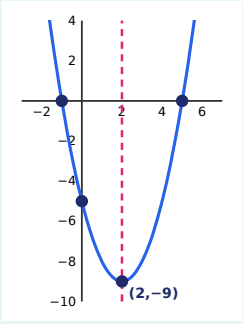
$a > 0$ opens up (minimum); $a < 0$ opens down (maximum).

3 KEY FEATURES OF A PARABOLA

Axis: $x = -\dfrac{b}{2a}$. Substitute it to find the vertex.

Example: $y = x^2 - 4x - 5$
Axis: $x = \frac{4}{2} = 2$
Vertex: $(2, -9)$
Zeros: $(x - 5)(x + 1) = 0 \to -1,\ 5$

Answer: vertex $(2, -9)$, y-intercept $(0, -5)$

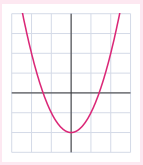


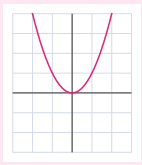
4 VERTEX FORM & TRANSFORMATIONS

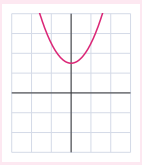
h in $y = a(x - h)^2 + k$	**shift right** h
k	**shift up** k
$\lvert a\rvert > 1$ or $\lvert a\rvert < 1$	**narrower or wider**
$a < 0$	**opens down**
$y = -2(x + 4)^2 + 5$	**vertex** $(-4, 5)$

5 THE DISCRIMINANT

In $ax^2 + bx + c = 0$, the discriminant $b^2 - 4ac$ tells how many real solutions.


2 x-intercepts


1 x-intercept


no x-intercepts

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Write $= 0$ before solving.
- Square roots give two answers: $\pm$.
- The y-intercept is c; the zeros are where $y = 0$.

✗ Common mistakes

- $x = \frac{b}{2a}$: the axis is $-\frac{b}{2a}$
- In $(x - 3)^2$, $h = -3$: it is $h = 3$
- Dividing only part of the formula by $2a$: divide all of $-b \pm \sqrt{}$

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More practice: solving quadratic equations: factoring, square roots, completing the square and the formula

1 SOLVE BY FACTORING

Set equal to 0, factor, use the Zero Product Property.

Example: $x^2 - 5x - 6 = 0$
Factor: $(x - 6)(x + 1) = 0$
Answer: $x = 6$ **or** $x = -1$

2 SOLVE BY SQUARE ROOTS

Isolate the square, then take $\pm\sqrt{}$. Simplify radicals.

Example: $(x - 3)^2 = 20$
Square root: $x - 3 = \pm\sqrt{20} = \pm 2\sqrt{5}$
Answer: $x = 3 \pm 2\sqrt{5}$

3 COMPLETING THE SQUARE

Move c, add $\left(\frac{b}{2}\right)^2$ to both sides, write a square, solve.

Example: $x^2 + 6x - 7 = 0$
Move 7: $x^2 + 6x = 7$
Add 9: $(x + 3)^2 = 16$
Roots: $x + 3 = \pm 4$
Answer: $x = 1$ **or** $x = -7$

4 QUADRATIC FORMULA

For $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: $2x^2 + 4x - 6 = 0$
a, b, c: $2,\ 4,\ -6$
Substitute: $\dfrac{-4 \pm \sqrt{16 + 48}}{4} = \dfrac{-4 \pm 8}{4}$
Answer: $x = 1$ **or** $x = -3$

5 CHOOSING A METHOD

No x-term: $x^2 = k$ or $(x - h)^2 = k$	**square roots**
Factors easily	**factoring**
$a = 1$ and b is even	**completing the square**
Anything else (always works)	**quadratic formula**

6 PROJECTILE MOTION

$h(t) = -16t^2 + v_0 t + h_0$ (feet). Vertex = maximum height; positive zero = landing time.

Example: $h(t) = -16t^2 + 32t + 48$
Max at $t = 1$**:** $h(1) = 64$ ft
Lands: $-16(t - 3)(t + 1) = 0 \rightarrow t = 3$
Answer: 64 ft high, lands at 3 s