Quadratic Functions & Complex Numbers Error Analysis

Find the mistake. Explain it. Fix it.

Name: _____ Period: _____ Date: _____

1. Vertex form: sign of *h*

ORIGINAL PROBLEM

Identify the vertex and the axis of symmetry of $f(x) = -(x + 2)^2 + 3$.

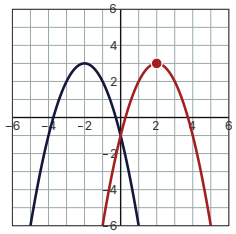
INCORRECT WORKED SOLUTION

Vertex form: $f(x) = a(x - h)^2 + k$

$f(x) = -(x + 2)^2 + 3$, so $h = 2$ and $k = 3$.

Vertex: (2, 3)

Axis of symmetry: $x = 2$

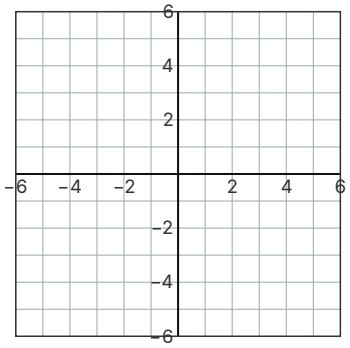


Navy: $f(x)$. Red: parabola with the student's vertex (2, 3).

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

2. Completing the square

ORIGINAL PROBLEM

Write $f(x) = 2x^2 - 12x + 7$ in vertex form. Then state the minimum value.

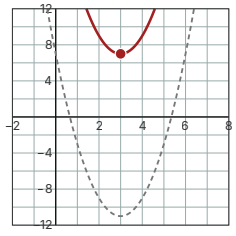
INCORRECT WORKED SOLUTION

$f(x) = 2(x^2 - 6x) + 7$

Half of -6 is -3, and $(-3)^2 = 9$:

$f(x) = 2(x^2 - 6x + 9) + 7$

$f(x) = 2(x - 3)^2 + 7$. Minimum value: 7

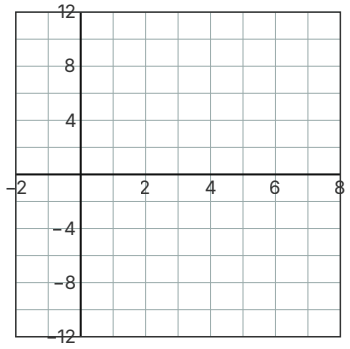


Dashed: $f(x)$. Red: the student's vertex form.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

Scoring: Explanation = 1 point • Correct work/answer = 1 point • Total = 20 points

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3. Square roots: the ± step

ORIGINAL PROBLEM

Solve $3(x + 1)^2 - 12 = 0$.

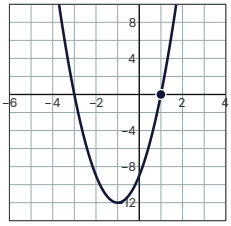
INCORRECT WORKED SOLUTION

$3(x + 1)^2 = 12$

$(x + 1)^2 = 4$

$x + 1 = 2$

$x = 1$

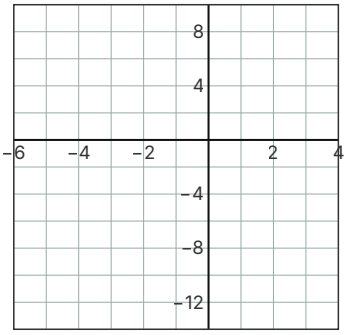


The graph of $y = 3(x + 1)^2 - 12$. Dot: the student's solution.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

4. Quadratic formula: dividing the numerator

ORIGINAL PROBLEM

Solve $3x^2 - 6x - 1 = 0$ with the quadratic formula.

INCORRECT WORKED SOLUTION

$a = 3, b = -6, c = -1$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)} = \frac{6 \pm \sqrt{48}}{6}$$

$\sqrt{48} = 4\sqrt{3}$, so $x = \dfrac{6 \pm 4\sqrt{3}}{6}$

Divide by 6: $x = 1 \pm 4\sqrt{3}$

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)

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5. Discriminant with a negative *b*

ORIGINAL PROBLEM

Use the discriminant to decide how many real solutions $x^2 - 6x + 5 = 0$ has.

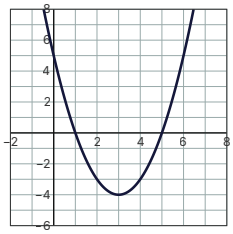
INCORRECT WORKED SOLUTION

$a = 1, b = -6, c = 5$

$b^2 - 4ac = -6^2 - 4(1)(5)$

$= -36 - 20 = -56$

The discriminant is negative, so there are no real solutions.

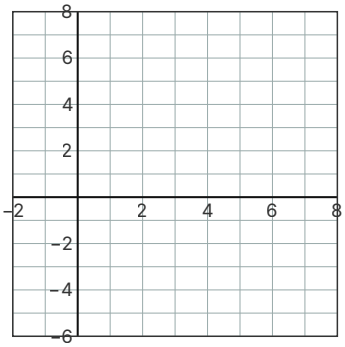


The graph of $y = x^2 - 6x + 5$.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

6. Powers of *i*

ORIGINAL PROBLEM

Multiply $(2 + 3i)(1 - 4i)$ and write the product in the form $a + bi$.

INCORRECT WORKED SOLUTION

$2(1) + 2(-4i) + 3i(1) + 3i(-4i)$

$= 2 - 8i + 3i - 12i^2$

$= 2 - 5i - 12$

$= -10 - 5i$

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)

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7. Dividing complex numbers

ORIGINAL PROBLEM

Write $\frac{5 + 2i}{3 - i}$ in the form $a + bi$.

INCORRECT WORKED SOLUTION

Multiply the top and bottom by $3 - i$:

$$\frac{(5 + 2i)(3 - i)}{(3 - i)(3 - i)} = \frac{15 - 5i + 6i - 2i^2}{9 - 3i - 3i + i^2}$$

$$= \frac{17 + i}{8 - 6i}$$

Answer: $\frac{17 + i}{8 - 6i}$

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)

8. Quadratic inequality: which region?

ORIGINAL PROBLEM

Solve $x^2 - 2x - 8 > 0$. Write the solution in interval notation.

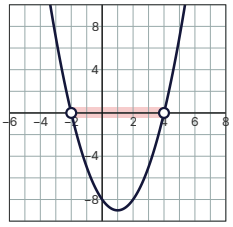
INCORRECT WORKED SOLUTION

Factor: $(x - 4)(x + 2) > 0$

Critical values: $x = 4$ and $x = -2$

Take the values between them: $-2 < x < 4$

Solution: $(-2, 4)$

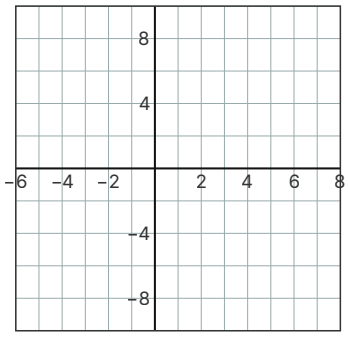


Bar: the student's solution, on the graph of $y = x^2 - 2x - 8$.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

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9. Quadratic inequality: shading

ORIGINAL PROBLEM

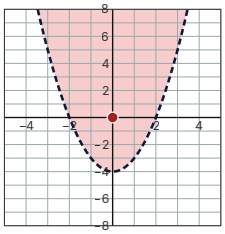
Graph $y < x^2 - 4$ and shade the solution set.

INCORRECT WORKED SOLUTION

Boundary: $y = x^2 - 4$, dashed because the inequality is strict.

The parabola opens up, so shade inside the U.

Shade the region above the curve.

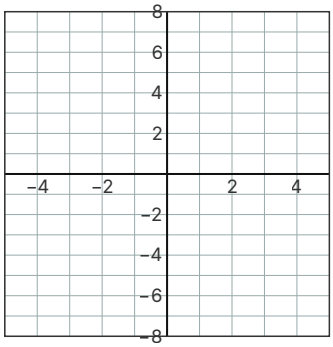


Red region: student's. (0, 0) fails $y < x^2 - 4$.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

10. Maximum value of a quadratic

ORIGINAL PROBLEM

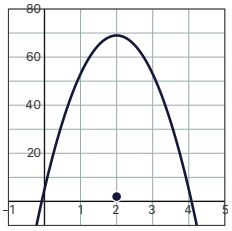
A ball is thrown upward. Its height is $h(t) = -16t^2 + 64t + 5$, where t is in seconds and h is in feet. Find the maximum height.

INCORRECT WORKED SOLUTION

The maximum occurs at the vertex.

$$t = -\frac{b}{2a} = -\frac{64}{2(-16)} = 2$$

The maximum height is 2 feet.

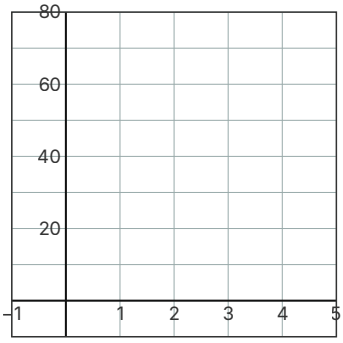


Dot: where the student's answer puts the ball at $t = 2$.

A. EXPLAIN THE MISTAKE (1 POINT)

Describe exactly which step is wrong and why.

B. SHOW CORRECT WORK (1 POINT)



Sketch the correct graph on this grid.

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