Unit 3 · Quadratics & Complex Numbers Cheat Sheet

Unit 3 focus: parabola forms, completing the square, the quadratic formula and the discriminant

1 KEY VOCABULARY

vertex	Turning point (h, k): min if $a > 0$, max if $a < 0$.	$(3, -8)$
axis of symmetry	Vertical line $x = h$ through the vertex.	$x = 3$
discriminant	$b^2 - 4ac$: number and type of solutions.	40
imaginary unit	$i = \sqrt{-1}$, so $i^2 = -1$.	$\sqrt{-49} = 7i$
complex conjugate	$a - bi$ for $a + bi$; product $a^2 + b^2$.	$(3 + 2i)(3 - 2i) = 13$

2 THREE FORMS

Form	Equation	Shows
Vertex	$a(x - h)^2 + k$	vertex (h, k)
Standard	$ax^2 + bx + c$	axis $-\frac{b}{2a}$, y-int c
Factored	$a(x - p)(x - q)$	zeros p, q

$2x^2 - 12x + 10 = 2(x - 3)^2 - 8 = 2(x - 1)(x - 5)$

3 COMPLETING THE SQUARE

Add $\left(\frac{b}{2}\right)^2$ to both sides (divide by a first in an equation).

Example: Solve $x^2 - 6x + 3 = 0$.
Move 3, add 9: $(x - 3)^2 = 6$
Roots: $x - 3 = \pm\sqrt{6}$
Answer: $x = 3 \pm \sqrt{6}$

4 QUADRATIC FORMULA

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Divide every term by $2a$.

Example: $3x^2 - 4x - 2 = 0$
Discriminant: $16 + 24 = 40$
Formula: $\dfrac{4 \pm 2\sqrt{10}}{6}$
Answer: $x = \dfrac{2 \pm \sqrt{10}}{3}$

5 THE DISCRIMINANT

$b^2 - 4ac$	Solutions	Example
positive square	2 rational	$x^2 - 2x - 3$: 16
positive, not square	2 irrational	$x^2 - 2x - 1$: 8
zero	1 real (double)	$x^2 - 4x + 4$: 0
negative	2 complex conjugates	$x^2 - 4x + 6$: -8

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- In $(x + 4)^2$ the vertex has $x = -4$.
- Square roots give $\pm$.
- Factor out a before completing the square in vertex form.

✗ Common mistakes

- b^2 for $b = -5$ is not -25: it is $(-5)^2 = 25$
- $\frac{4+2\sqrt{10}}{6} \neq 4 + \frac{\sqrt{10}}{3}$: divide both terms
- $i^2 = 1$: it is -1

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More practice: complex numbers, quadratic inequalities, linear-quadratic systems and modeling

1 POWERS OF THE IMAGINARY UNIT

i^1	i^2	i^3	i^4
i	-1	$-i$	1

The cycle repeats every 4: use the remainder after dividing the exponent by 4. i^{47}: $47 = 4(11) + 3$, so $i^{47} = -i$. $\sqrt{-18} = 3i\sqrt{2}$.

2 COMPLEX NUMBER OPERATIONS

Add: $(3 + 2i) + (5 - 6i)$	$8 - 4i$
Subtract: $(3 + 2i) - (5 - 6i)$	$-2 + 8i$
Multiply: $(3 + 2i)(4 - i) = 12 + 5i - 2i^2$	$14 + 5i$
Divide: $\frac{8+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{15+10i}{5}$	$3 + 2i$
$\lvert 3 + 4i \rvert = \sqrt{9 + 16}$	5

3 COMPLEX ROOTS

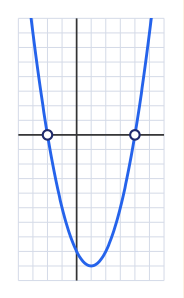
A negative discriminant gives conjugate roots. Write a quadratic from roots: $a(x - p)(x - q)$.

Example: Solve $x^2 - 4x + 13 = 0$.
Discriminant: $16 - 52 = -36$
Formula: $\dfrac{4 \pm 6i}{2}$
Answer: $x = 2 \pm 3i$

4 QUADRATIC INEQUALITIES

Solve the equation, sketch, and test a point in each region.

Example: $x^2 - 2x - 8 > 0$
Zeros: $(x - 4)(x + 2) = 0 \to -2,\ 4$
Opens up: positive outside the zeros
Answer: $(-\infty, -2) \cup (4, \infty)$



5 LINEAR-QUADRATIC SYSTEMS

Substitute, get a quadratic = 0, solve, then find each y. 2, 1 or 0 intersection points.

Example: $y = x^2 - 4x + 3$ and $y = x - 1$
Set equal: $x^2 - 5x + 4 = 0$
Factor: $x = 1,\ 4$
Answer: $(1, 0)$ **and** $(4, 3)$

6 QUADRATIC MODELING

Vertex = maximum (the h-value is the max, t is when). Reject negative times.

Example: $h(t) = -16t^2 + 64t + 80$
Max at $t = 2$: $h(2) = 144$ ft
Lands: $-16(t - 5)(t + 1) = 0 \to t = 5$
Answer: 144 ft at 2 s; lands at 5 s