Unit 11 · Probability & Statistics Cheat Sheet

Unit 11 focus: counting, permutations, combinations, independent and dependent events, and conditional probability

1 KEY VOCABULARY

sample space	The set of all possible outcomes.	{H, T}
permutation	An arrangement where order matters.	president, VP
combination	A selection where order does not matter.	a 3-person team
independent	One event does not change the other's probability.	two dice
conditional probability	$P(A \mid B)$: probability of A given B happened.	$\frac{P(A \cap B)}{P(B)}$

2 COUNTING & PERMUTATIONS

Counting principle: 4 shirts, 3 pants, 2 shoes	$4 \cdot 3 \cdot 2 = 24$
${}_nP_r = \frac{n!}{(n-r)!}$: 1st, 2nd, 3rd from 8 runners	${}_8P_3 = 336$
Arrange all letters of BOOK (O repeats)	$\frac{4!}{2!} = 12$
Seat 5 people in a row	$5! = 120$

3 COMBINATIONS

${}_nC_r = \frac{n!}{r!(n-r)!}$. Multiply when choosing from separate groups.

Example: A committee of 3 from 7 boys and 5 girls with exactly 2 girls.

Girls: ${}_5C_2 = 10$

Boys: ${}_7C_1 = 7$

Answer: $10 \cdot 7 = 70$ **committees**

4 INDEPENDENT & DEPENDENT

Independent: $P(A \text{ and } B) = P(A) \cdot P(B)$. Two heads in a row	$\frac{1}{4}$
Dependent: 2 red without replacement from 5 red, 3 blue: $\frac{5}{8} \cdot \frac{4}{7}$	$\frac{5}{14}$
Or: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. King or heart	$\frac{4+13-1}{52} = \frac{4}{13}$

5 CONDITIONAL PROBABILITY

	Sport	No sport	Total
Juniors	18	12	30
Seniors	20	10	30
Total	38	22	60

$P(\text{sport} \mid \text{junior}) = \frac{18}{30} = \frac{3}{5}$, but $P(\text{junior} \mid \text{sport}) = \frac{18}{38} = \frac{9}{19}$. The condition is the denominator.

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Ask "does order matter?" before choosing P or C.
- Probabilities are between 0 and 1 and the sample space sums to 1.
- "At least one" = $1 - P(\text{none})$.

✘ Common mistakes

- $P(A \mid B) = P(B \mid A)$: usually different denominators
- Adding for "and": multiply for "and", add for "or"
- Forgetting to subtract the overlap: $P(A \text{ or } B)$ subtracts $P(A \text{ and } B)$

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More practice: probability distributions, expected value, normal distributions, z-scores and sampling

1 PROBABILITY DISTRIBUTIONS

Heads in 3 flips X	**0**	**1**	**2**	**3**
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Each $P \geq 0$ and they sum to 1. Binomial: $P(k) = {}_nC_k\, p^k (1-p)^{n-k}$. Exactly 4 of 5 free throws at $p = 0.7$: $5(0.7)^4(0.3) \approx 0.360$.

2 EXPECTED VALUE

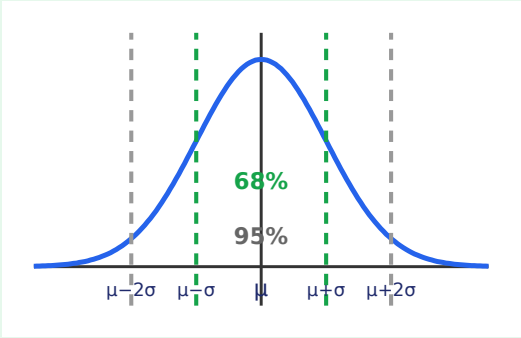
$E(X) = \sum x \cdot P(x)$: the long-run average per trial. A fair game has $E = 0$.

Example: Pay \$2 to roll a die; win \$10 on a 6.
Net: $+8$ with $\frac{1}{6}$, -2 with $\frac{5}{6}$
E: $\frac{8}{6} - \frac{10}{6} = -\frac{1}{3}$

Answer: lose about \$0.33 per game

3 NORMAL DISTRIBUTIONS

Bell-shaped and symmetric about the mean μ. Empirical rule: about **68%** within 1σ, **95%** within 2σ, **99.7%** within 3σ.



4 Z-SCORES

$z = \dfrac{x - \mu}{\sigma}$: how many standard deviations from the mean.

Example: Scores: $\mu = 70$, $\sigma = 8$. A student scores 86.
z: $\frac{86-70}{8} = 2$
Rule: 95% within ± 2, so 2.5% above

Answer: about 97.5% scored lower

5 SAMPLING & STUDIES

Survey	asks a sample; random sampling avoids bias
Observational study	observes without assigning treatments: no cause and effect
Experiment	randomly assigns treatments: can show cause and effect
Margin of error	$\approx \pm\frac{1}{\sqrt{n}}$: $n = 400$ gives $\pm 5\%$

6 USING A SAMPLE

A sample proportion estimates the population proportion; report it with its margin of error.

Example: 240 of 400 randomly chosen voters support a measure.
Proportion: $\frac{240}{400} = 60\%$
Interval: $60\% \pm 5\%$

Answer: likely between 55% and 65%