Unit 7 · Vectors Cheat Sheet

Unit 7 focus: vector components, magnitude, direction and operations

1 KEY VOCABULARY

vector	Has magnitude and direction.	$3, 4$
magnitude	Length of the vector.	$\|\|\langle 3, 4\rangle\|\| = 5$
unit vector	Magnitude 1.	$\langle \frac{3}{5}, \frac{4}{5} \rangle$
resultant	The sum of vectors.	u + v

2 COMPONENT FORM

Terminal point minus initial point.

Example: From P(1, −2) to Q(6, 10)
Subtract: $6 - 1, 10 - (-2)$
Answer: $5, 12$**, magnitude 13**

3 MAGNITUDE AND DIRECTION

$\|v\| = \sqrt{a^2 + b^2}$, $\tan\theta = \frac{b}{a}$ (check the quadrant).

Example: v = $-3, 3$
Magnitude: $\sqrt{18} = 3\sqrt{2}$
Direction: quadrant II
Answer: $3\sqrt{2}$**, 135°**

4 VECTOR OPERATIONS

Add or scale component by component.

Example: u = $2, -1$, v = $-4, 3$. Find 3u − v.
3u: $6, -3$
Subtract v: $6 + 4, -3 - 3$
Answer: $10, -6$

5 VECTOR FROM MAGNITUDE AND ANGLE

$v = \langle \|v\| \cos\theta, \|v\| \sin\theta \rangle$

Example: |v| = 10 at 30°
Components: $10cos30^\circ, 10sin30^\circ$
Answer: $\langle 5\sqrt{3}, 5 \rangle$

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Component form = terminal − initial.
- Use atan2 thinking: check the quadrant.
- Magnitude is never negative.

✘ Common mistakes

- Initial − terminal: terminal − initial
- $\tan^{-1}(3 \div -3) = -45°$: 135° (quadrant II)
- |u + v| = |u| + |v|: only if parallel

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More practice: dot product, angles between vectors, projections and applications

1 DOT PRODUCT

$u \cdot v = u_1v_1 + u_2v_2$ (a number).

Example: u = $3, 4$, v = $2, -1$
Multiply and add: $6 - 4$
Answer: 2

2 ANGLE BETWEEN VECTORS

$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$

Example: u = $1, 0$, v = $1, 1$
cos θ: $\frac{1}{1 \cdot \sqrt{2}}$
Answer: 45°

3 ORTHOGONAL AND PARALLEL

Test	Meaning
u · v = 0	orthogonal (perpendicular)
u = kv	parallel
u · v > 0	angle < 90°

$2, 3 \cdot -3, 2 = 0$, so they are orthogonal.

4 PROJECTION

$\text{proj}_v u = \frac{u \cdot v}{\|v\|^2} v$

Example: u = $4, 2$ onto v = $3, 0$
Scalar: $\frac{12}{9} = \frac{4}{3}$
Answer: $4, 0$

5 WORK

W = F · d = $\|F\|\|d\| \cos\theta$

Example: A 50 N force at 60° moves a box 10 m.
Compute: 50 · 10 · cos 60°
Answer: 250 J

6 CHECK YOURSELF

- The dot product is a NUMBER, not a vector.
- Did I divide by $|v|^2$ for the projection?
- Are my angles in the right quadrant?
- Did I sketch the vectors?