Unit 9 · Sequences, Series, and Limits Cheat Sheet

Unit 9 focus: arithmetic and geometric sequences, finite series and sigma notation

1 ARITHMETIC VS GEOMETRIC

Type	nth term
Arithmetic: add the same d	$a_n = a_1 + (n-1)d$
Geometric: multiply by the same r	$a_n = a_1 r^{n-1}$
Test	equal differences → arithmetic; equal ratios → geometric

2 ARITHMETIC NTH TERM

Find d, then use n − 1 steps.

Example: 7, 11, 15, 19, … Find the 30th term.
d = 4: $a_{30} = 7 + 29 \cdot 4$

Answer: 123

3 GEOMETRIC NTH TERM

Find r by dividing a term by the one before it.

Example: 3, 6, 12, 24, … Find the 8th term.
r = 2: $a_8 = 3 \cdot 2^7$

Answer: 384

4 ARITHMETIC SERIES

Average of the first and last term, times the number of terms.

Example: Sum of the first 20 terms of 5 + 8 + 11 + …
Last term: $a_{20} = 5 + 19 \cdot 3 = 62$
Sum: $S_{20} = \frac{20(5+62)}{2}$

Answer: 670

5 GEOMETRIC SERIES

$S_n = \frac{a_1(1-r^n)}{1-r}$, r ≠ 1.

Example: Sum of the first 6 terms of 2 + 6 + 18 + …
r = 3: $S_6 = \frac{2(1-3^6)}{1-3}$

Answer: 728

6 SIGMA NOTATION

Substitute each k from the bottom value to the top value and add.

Example: $\sum_{k=1}^{5}(2k+1)$
Terms: 3 + 5 + 7 + 9 + 11

Answer: 35

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More practice: infinite geometric series and an introduction to limits

1 INFINITE GEOMETRIC SERIES

Converges only when |r| < 1: $S = \frac{a_1}{1-r}$.

Example: 12 + 6 + 3 + 1.5 + ...
r = 1/2: $S = \frac{12}{1-1/2}$
Answer: 24

2 CONVERGE OR DIVERGE?

r = 2/3	**converges**
r = −1/2	**converges (terms alternate)**
r = 3/2	**diverges**
r = −1	**diverges**
arithmetic, d ≠ 0	**diverges**

3 LIMITS BY FACTORING

If substitution gives 0/0, factor, cancel, then substitute.

Example: $\lim_{x\to 3} \frac{x^2-9}{x-3}$
Factor: $\frac{(x-3)(x+3)}{x-3} = x+3$
Substitute: 3 + 3
Answer: 6

4 LIMITS AT INFINITY

Degrees of top and bottom	Limit as x → ∞
top < bottom	0
top = bottom	ratio of leading coefficients
top > bottom	∞ or −∞ (no finite limit)

5 CONJUGATES AND ONE-SIDED LIMITS

Multiply by the conjugate to clear a square root.

Example: $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$
Conjugate: $\frac{1}{\sqrt{x+4}+2}$
Substitute: $\frac{1}{2+2}$
Answer: 1/4

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Check differences and ratios before choosing a formula.
- Use n − 1 steps from the first term to the nth term.
- A limit exists only if both one-sided limits agree.

✗ Common mistakes

- $a_1/(1 - r)$ with r = 2: diverges
- 0/0 → no limit: factor first
- Σ k = 2 to 10 → 10 terms: 9 terms