Unit 8 · Probability Cheat Sheet

Unit 8 focus: counting, permutations, combinations and basic probability

1 FUNDAMENTAL COUNTING PRINCIPLE

Multiply the number of choices at each stage.

Example: A code: 2 letters then 3 digits
Multiply: $26^2 \cdot 10^3$
Answer: 676,000

2 PERMUTATIONS AND COMBINATIONS

Type	Formula
Permutation (order matters)	${}_nP_r = \frac{n!}{(n-r)!}$
Combination (order does not)	${}_nC_r = \frac{n!}{r!(n-r)!}$
Repeated letters	$\frac{n!}{n_1!\,n_2!\cdots}$

3 PERMUTATIONS

Order matters: president, VP, treasurer.

Example: Choose 3 officers from 10 people.
Compute: $10 \cdot 9 \cdot 8$
Answer: 720

4 COMBINATIONS

Order does not matter: a committee.

Example: Choose a 3-person committee from 10.
Divide by 3!: $\frac{720}{6}$
Answer: 120

5 PROBABILITY RULES

Rule	Formula
Complement	P(A') = 1 − P(A)
Addition	P(A or B) = P(A) + P(B) − P(A and B)
Mutually exclusive	P(A and B) = 0

Draw a card: P(heart or king) = 13/52 + 4/52 − 1/52 = 16/52.

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Ask: does order matter?
- 0! = 1.
- Probabilities are between 0 and 1.

✘ Common mistakes

- Using P for a committee: C (order does not matter)
- P(A or B) = P(A) + P(B) always: subtract the overlap
- P(at least one) by listing all cases: 1 − P(none)

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More practice: conditional probability, independence, the binomial theorem and expected value

1 CONDITIONAL PROBABILITY

$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$

Example: P(B) = 0.5, P(A and B) = 0.2
Divide: $\frac{0.2}{0.5}$
Answer: 0.4

2 INDEPENDENT EVENTS

A and B independent ⇔ P(A and B) = P(A)P(B).

Example: P(A) = 0.3, P(B) = 0.6, P(A and B) = 0.18
Check: 0.3 · 0.6 = 0.18 ✓
Answer: Independent

3 AT LEAST ONE

Use the complement.

Example: P(at least one 6 in 3 rolls)
Complement: $1 - (\frac{5}{6})^3$
Answer: $\frac{91}{216} \approx 0.421$

4 BINOMIAL PROBABILITY

$P(X = k) = {}_nC_k\, p^k (1-p)^{n-k}$

Example: Exactly 2 heads in 5 fair flips
Compute: $10 \cdot (\frac{1}{2})^5$
Answer: $\frac{10}{32} = \frac{5}{16}$

5 BINOMIAL THEOREM

Term k + 1 of $(a+b)^n$: ${}_nC_k\, a^{n-k} b^k$

Example: Coefficient of x^2 in $(x+3)^4$
k = 2: ${}_4C_2 \cdot 3^2 = 6 \cdot 9$
Answer: 54

6 EXPECTED VALUE

E(X) = Σ x · P(x).

Example: Win $10 with P = 0.2, lose $3 with P = 0.8.
Compute: 10(0.2) + (−3)(0.8)
Answer: −$0.40 per play