Unit 2 · Functions Cheat Sheet

Unit 2 focus: domain, range, graphs and transformations

1 KEY VOCABULARY

domain	All allowed inputs.	$\sqrt{x-2}$: x ≥ 2
range	All outputs.	x^2: y ≥ 0
even function	f(−x) = f(x); y-axis symmetry.	x^2, cos x
odd function	f(−x) = −f(x); origin symmetry.	x^3, sin x

2 TRANSFORMATIONS OF F(X)

Form	Effect
f(x) + k	up k
f(x − h)	right h
−f(x)	reflect over x-axis
f(−x)	reflect over y-axis
a·f(x)	vertical stretch by a
f(bx)	horizontal shrink by $\frac{1}{b}$

3 COMPOSITION

(f ∘ g)(x) = f(g(x)): inside first.

Example: $f(x) = x^2 + 1$, $g(x) = 3x - 2$; find (f ∘ g)(2).
Inside: g(2) = 4
Outside: f(4) = 17
Answer: 17

4 INVERSE FUNCTIONS

Swap x and y, then solve for y.

Example: $f(x) = \frac{2x+1}{3}$
Swap: 3x = 2y + 1
Answer: $f^{-1}(x) = \frac{3x-1}{2}$

5 AVERAGE RATE OF CHANGE

$\frac{f(b)-f(a)}{b-a}$

Example: $f(x) = x^3$ on [1, 3]
Compute: $\frac{27-1}{2}$
Answer: 13

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- f(x − h) moves RIGHT, not left.
- $f^{-1}(x) \neq \frac{1}{f(x)}$.
- Only one-to-one functions have inverses.

✘ Common mistakes

- (f ∘ g)(x) = f(x) · g(x): plug g into f
- f(x + 3) shifts right 3: left 3
- Domain of $\frac{1}{x-4}$ is all reals: x ≠ 4

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More practice: combinations, inverses, polynomial, rational, exponential and logarithmic functions

1 POLYNOMIAL END BEHAVIOR

Degree · lead coefficient	Ends
even · positive	both up
even · negative	both down
odd · positive	down left, up right
odd · negative	up left, down right

A degree-n polynomial has at most n − 1 turning points.

2 RATIONAL FUNCTION ASYMPTOTES

Compare degrees n (top), d (bottom)	Horizontal asymptote
n < d	y = 0
n = d	y = ratio of leading coefficients
n = d + 1	slant asymptote

Vertical asymptotes: zeros of the reduced denominator.
For $\frac{2x+1}{x-3}$: x = 3, y = 2.

3 LOG PROPERTIES

Property	Rule
Product	$\log_b(MN) = \log_b M + \log_b N$
Quotient	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power	$\log_b M^p = p \log_b M$
Change of base	$\log_b M = \frac{\ln M}{\ln b}$

4 EXPONENTIAL EQUATIONS

Match bases, or take a log of both sides.

Example: $5^{x-1} = 125$
Match bases: $5^{x-1} = 5^3$
Answer: x = 4

5 LOGARITHMIC EQUATIONS

Rewrite in exponential form; check the domain.

Example: $\log_2(x+3) = 5$
Exponential form: $x + 3 = 2^5$
Answer: x = 29

6 EXPONENTIAL GROWTH MODEL

$A = Pe^{rt}$ for continuous growth.

Example: $2,000 at 4% compounded continuously for 5 years
Compute: $2000e^{0.2}$
Answer: ≈ $2,442.81