Unit 3 · Conics Cheat Sheet

Unit 3 focus: parabolas and circles

1 PARABOLAS

Form	Opens · focus
$(x-h)^2 = 4p(y-k)$	up/down · (h, k + p)
$(y-k)^2 = 4p(x-h)$	right/left · (h + p, k)

Vertex (h, k). The directrix is p units on the other side of the vertex.

2 PARABOLA FOCUS AND DIRECTRIX

Match 4p, then move p from the vertex.

Example: $x^2 = 12y$

4p = 12: p = 3

Answer: focus (0, 3), directrix y = −3

3 CIRCLES

Form	Gives
$(x-h)^2 + (y-k)^2 = r^2$	center (h, k), radius r

4 COMPLETE THE SQUARE

Group x's and y's, add the square of half each coefficient to both sides.

Example: $x^2 + y^2 - 6x + 4y - 12 = 0$

Group: $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$

Answer: center (3, −2), r = 5

5 KEY VOCABULARY

focus	Fixed point used to define a conic.	parabola: (0, p)
directrix	Fixed line for a parabola.	y = −p
vertex	Turning point / end of the major axis.	(h, k)
eccentricity	$e = \frac{c}{a}$: how stretched.	circle e = 0

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Signs inside the parentheses are opposite the center.
- Add the same amounts to BOTH sides when completing the square.
- r is the square root of the right side.

✘ Common mistakes

- (x + 3)² → center x = 3: x = −3
- Radius = 25: $r = \sqrt{25} = 5$
- Forgetting to add to the right side: balance both sides

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More practice: ellipses, hyperbolas and identifying conics

1 ELLIPSES

Form	Key facts
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	a > b: horizontal major axis
Foci	$c^2=a^2-b^2$
Lengths	major axis 2a, minor axis 2b

2 ELLIPSE FOCI

Find c from a and b.

Example: $\frac{x^2}{25}+\frac{y^2}{9}=1$
$c^2 = 25 - 9 = 16$: c = 4

Answer: foci (±4, 0), vertices (±5, 0)

3 HYPERBOLAS

Form	Key facts
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	opens left/right
Foci	$c^2=a^2+b^2$
Asymptotes	$y-k=\pm\frac{b}{a}(x-h)$

4 HYPERBOLA FACTS

Positive term tells the direction.

Example: $\frac{x^2}{9}-\frac{y^2}{16}=1$
$c^2 = 9 + 16 = 25$: c = 5

Answer: foci (±5, 0), asymptotes $y=\pm\frac{4}{3}x$

5 IDENTIFY THE CONIC

$Ax^2+Cy^2+\cdots=0$	Conic
A = C	circle
A·C > 0, A ≠ C	ellipse
A·C < 0	hyperbola
A = 0 or C = 0	parabola

$4x^2-y^2+8x=0$: A·C = −4 < 0 → hyperbola.

6 CHECK YOURSELF

- Ellipse: $c^2 = a^2 - b^2$. Hyperbola: $c^2 = a^2 + b^2$.
- Is a under the larger (ellipse) or positive (hyperbola) term?
- Did I complete the square on both variables?
- Did I sketch to check my answer?