Unit 7 · Geometry Cheat Sheet

Unit 7 focus: the Pythagorean theorem and its converse in 2-D, 3-D and on the coordinate plane

1 KEY VOCABULARY

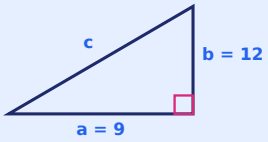
legs	The two sides that form the right angle.	a and b
hypotenuse	The longest side, across from the right angle.	c
Pythagorean triple	Whole-number sides of a right triangle.	3-4-5, 5-12-13, 8-15-17
converse	If $a^2 + b^2 = c^2$, the triangle is right.	6, 8, 10 is right

2 THE PYTHAGOREAN THEOREM

In a right triangle, $a^2 + b^2 = c^2$ (c is the hypotenuse).

Example: Legs 9 and 12. Find c.
Substitute: $9^2 + 12^2 = 81 + 144 = 225$
Square root: $c = \sqrt{225}$

Answer: $c = 15$



3 FINDING A LEG

Subtract: $a^2 = c^2 - b^2$.

Example: A 13-ft ladder reaches 12 ft up a wall. How far is its base from the wall?
Subtract: $a^2 = 13^2 - 12^2 = 169 - 144 = 25$

Answer: $a = 5$ **ft**

4 IS IT A RIGHT TRIANGLE?

Sides	Test $a^2 + b^2 \stackrel{?}{=} c^2$	Right?
7, 24, 25	$49 + 576 = 625$	yes
5, 9, 11	$25 + 81 = 106 \neq 121$	no
10, 24, 26	$100 + 576 = 676$	yes

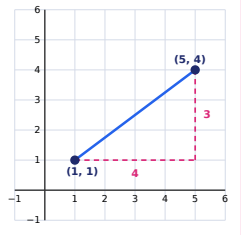
Always use the **longest** side as c.

5 DISTANCE BETWEEN POINTS

Draw a right triangle: the legs are the horizontal and vertical distances. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example: From $(1, 1)$ to $(5, 4)$
Legs: $5 - 1 = 4$ and $4 - 1 = 3$
Theorem: $d = \sqrt{16 + 9}$

Answer: $d = 5$



6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- The hypotenuse is always the longest side.
- Round only at the end; $\sqrt{50} \approx 7.07$.
- Draw a picture for word problems.

✘ Common mistakes

- $a + b = c$: square first: $a^2 + b^2 = c^2$
- Adding when finding a leg: subtract from c^2
- Stopping at $c^2 = 225$: take the square root

Unit 7 · Geometry Cheat Sheet

More practice: volume of cylinders, cones, spheres and composite solids

1 PYTHAGOREAN THEOREM IN 3-D

The diagonal of a box: use the theorem twice, or $d = \sqrt{\ell^2 + w^2 + h^2}$.

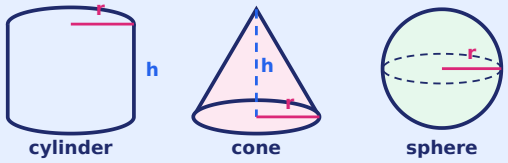
Example: A box is 3 by 4 by 12. Find the longest diagonal.
Base diagonal: $\sqrt{3^2 + 4^2} = 5$
Space diagonal: $\sqrt{5^2 + 12^2} = \sqrt{169}$

Answer: $d = 13$

2 VOLUME FORMULAS

Solid	Volume
cylinder	$V = \pi r^2 h$
cone	$V = \frac{1}{3}\pi r^2 h$
sphere	$V = \frac{4}{3}\pi r^3$

A cone holds $\frac{1}{3}$ of the cylinder with the same base and height. Answers are in cubic units.



3 VOLUME EXAMPLES

Solid	Work	Volume
cylinder $r = 3, h = 5$	$\pi(9)(5)$	$45\pi \approx 141.4$
cone $r = 3, h = 5$	$\frac{1}{3}\pi(9)(5)$	$15\pi \approx 47.1$
sphere $r = 3$	$\frac{4}{3}\pi(27)$	$36\pi \approx 113.1$

Leave answers in terms of π or round as the problem asks. Diameter 10 means $r = 5$.

4 SOLVING FOR A MISSING DIMENSION

Substitute what you know into the formula, then solve.

Example: A cylinder has volume 72π and radius 3. Find the height.
Formula: $72\pi = \pi(3)^2 h = 9\pi h$
Divide by 9π: $h = 8$

Answer: $h = 8$

5 COMPOSITE SOLIDS

Split into familiar solids. Add the parts (or subtract a hole).

Example: An ice-cream cone: a cone ($r = 2, h = 6$) topped with a half sphere ($r = 2$).
Cone: $\frac{1}{3}\pi(4)(6) = 8\pi$
Half sphere: $\frac{1}{2} \cdot \frac{4}{3}\pi(8) = \frac{16}{3}\pi$

Answer: $\frac{40}{3}\pi \approx 41.9$ **cubic units**

6 CHECK YOURSELF

- Is c the longest side (across from the right angle)?
- Did I use the radius, not the diameter?
- Did I cube the radius for a sphere and square it for a cylinder or cone?
- Are my units cubic for volume?