Unit 10 · Factoring and Zero Product Cheat Sheet

Unit 10 focus: factoring GCF, trinomials, special patterns and grouping

1 KEY VOCABULARY

factor	Write as a product (undo distributing).	$6x + 9 = 3(2x + 3)$
GCF	Greatest common factor of every term.	$3x^2$ in $6x^3 + 9x^2$
trinomial	A polynomial with three terms.	$x^2 + 7x + 12$
prime	Cannot be factored over the integers.	$x^2 + 4$
zero / root	An x that makes the expression 0.	$x = -3$

2 FACTOR OUT THE GCF

Find the largest number and the lowest power of each variable that divide every term.

Example: Factor $6x^3 + 9x^2$.
GCF: $3x^2$
Divide each term: $2x$ and 3
Answer: $3x^2(2x + 3)$

3 TRINOMIALS, LEADING COEFFICIENT 1

Find two numbers that **multiply to c** and **add to b**.

Example: Factor $x^2 + 7x + 12$.
Multiply to 12, add to 7: 3 and 4
Answer: $(x + 3)(x + 4)$

4 TRINOMIALS: THE AC METHOD

AC method: find two numbers that multiply to ac and add to b, split the middle term, then group.

Example: Factor $2x^2 + 7x + 3$.
$ac = 6$: $6 \cdot 1$, sum 7
Split: $2x^2 + 6x + x + 3$
Group: $2x(x + 3) + 1(x + 3)$
Answer: $(2x + 1)(x + 3)$

5 SPECIAL PATTERNS

Pattern	Factors	Example
$a^2 - b^2$	$(a - b)(a + b)$	$x^2 - 25 = (x - 5)(x + 5)$
$a^2 + 2ab + b^2$	$(a + b)^2$	$x^2 + 8x + 16 = (x + 4)^2$
$a^2 - 2ab + b^2$	$(a - b)^2$	$x^2 - 6x + 9 = (x - 3)^2$

A **sum** of squares like $x^2 + 25$ is prime.

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Always take out the GCF first.
- Check signs: $c > 0$ → same signs; $c < 0$ → different signs.
- Multiply the factors back to check.

✘ Common mistakes

- Right product, wrong sum: both conditions must hold
- $x^2 - 9 \neq (x - 3)^2$: it is $(x - 3)(x + 3)$
- Stopping too early: factor completely

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More practice: factoring completely and solving quadratics with the Zero Product Property

1 FACTORING BY GROUPING

Four terms: group in pairs, factor each pair, then factor out the common binomial.

Example: Factor $x^3 + 3x^2 + 2x + 6$.
Group: $x^2(x + 3) + 2(x + 3)$
Answer: $(x + 3)(x^2 + 2)$

2 FACTOR COMPLETELY

GCF first, then look again at what is left.

Example: Factor $3x^2 - 12$.
GCF: $3(x^2 - 4)$
Difference of squares: $3(x - 2)(x + 2)$
Answer: $3(x - 2)(x + 2)$

3 STRATEGY

1. Is there a GCF?	**factor it out**
2. Two terms: $a^2 - b^2$?	**difference of squares**
3. Three terms: perfect square or $x^2 + bx + c$ / $ax^2 + bx + c$?	**pattern or AC method**
4. Four terms?	**grouping**
5. Can any factor be factored again?	**keep going**

4 ZERO PRODUCT PROPERTY

If $ab = 0$, then $a = 0$ or $b = 0$. Set each factor equal to zero.

Example: Solve $(x - 4)(2x + 6) = 0$.
$x - 4 = 0$: $x = 4$
$2x + 6 = 0$: $x = -3$
Answer: $x = 4$ **or** $x = -3$

5 SOLVING BY FACTORING

Get 0 on one side, factor, then use the Zero Product Property.

Example: Solve $x^2 + 2x = 15$.
Standard form: $x^2 + 2x - 15 = 0$
Factor: $(x + 5)(x - 3) = 0$
Answer: $x = -5$ **or** $x = 3$

6 APPLICATIONS

Write the equation, solve by factoring, and reject answers that do not make sense.

Example: A rectangle is 3 ft longer than it is wide; its area is 40 ft². Width?
Equation: $w(w + 3) = 40$
Factor: $(w + 8)(w - 5) = 0$
Answer: 5 ft (reject -8)