Unit 4 · Polynomial Functions Cheat Sheet

Unit 4 focus: operations, factoring patterns, polynomial division and the remainder and factor theorems

1 KEY VOCABULARY

degree	Highest exponent of the variable.	$4x^3 - x$: 3
leading coefficient	Coefficient of the highest-degree term.	$-2x^4 + x$: -2
zero (root)	An input with $P(a) = 0$; an x-intercept if real.	$x = 3$
multiplicity	How many times a factor repeats.	$(x-1)^2$: 2
end behavior	What $P(x)$ does as $x \to \pm\infty$.	up / down

2 OPERATIONS

Add: $(3x^3 - 2x + 5) + (x^3 + 4x^2 - 7)$	$4x^3 + 4x^2 - 2x - 2$
Subtract: $(2x^2 - x) - (5x^2 - 3x + 1)$	$-3x^2 + 2x - 1$
Multiply: $(x+3)(x^2 - 2x + 5)$	$x^3 + x^2 - x + 15$
Cube: $(x-2)^3$	$x^3 - 6x^2 + 12x - 8$

3 FACTORING PATTERNS

Sum of cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of cubes	$x^3 - 8 = (x-2)(x^2 + 2x + 4)$
Quadratic form	$x^4 - 5x^2 + 4 = (x-1)(x+1)(x-2)(x+2)$
Grouping	$x^3 + 2x^2 - 9x - 18 = (x+2)(x-3)(x+3)$

4 SYNTHETIC DIVISION

Divide by $x - a$: bring down, multiply by a, add. Write 0 for missing terms.

Example: $(x^3 - 4x^2 + x + 6) \div (x - 2)$
Row: $2 \mid 1 \quad -4 \quad 1 \quad 6 \to 1 \quad -2 \quad -3 \quad 0$
Quotient: $x^2 - 2x - 3$, remainder 0

Answer: $(x-2)(x-3)(x+1)$

5 REMAINDER & FACTOR THEOREMS

Remainder of $P(x) \div (x - a)$ is $P(a)$. $x - a$ is a factor exactly when $P(a) = 0$.

Example: $P(x) = 2x^3 - 3x^2 + 4x - 5$ divided by $x - 2$
Evaluate: $P(2) = 16 - 12 + 8 - 5$
Factor? no, since $P(2) \neq 0$

Answer: remainder 7

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Write terms in descending order before dividing.
- Use a, not $-a$, in synthetic division by $x - a$.
- Factor out a GCF first.

✗ Common mistakes

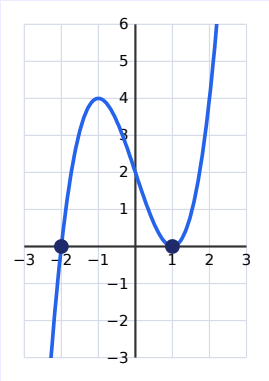
- $(x+2)^3 = x^3 + 8$: expand: $x^3 + 6x^2 + 12x + 8$
- Subtracting only the first term: distribute the minus sign to every term
- $x^3 + 8 = (x+2)^3$: it is $(x+2)(x^2 - 2x + 4)$

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More practice: zeros and multiplicity, end behavior, graphing, the Fundamental Theorem and applications

1 ZEROS & MULTIPLICITY

Odd multiplicity: the graph **crosses**. Even multiplicity: it **touches** and turns. $f(x) = (x+2)(x-1)^2$: crosses at -2, touches at 1.



2 END BEHAVIOR

Degree	$a_n > 0$	$a_n < 0$
even	up, up	down, down
odd	down left, up right	up left, down right

A degree-n polynomial has at most n real zeros and at most $n-1$ turning points. $-x^4 + 3x$: down, down.

3 GRAPHING A POLYNOMIAL

Factor, plot zeros, use end behavior and the y-intercept.

Example: $f(x) = x^3 - 4x$
Factor: $x(x-2)(x+2)$
Zeros: $-2, 0, 2$; odd degree, $a > 0$: down left, up right

Answer: crosses at $-2, 0, 2$



4 FUNDAMENTAL THEOREM OF ALGEBRA

Degree n means exactly n complex zeros (counting multiplicity). With real coefficients, complex zeros come in conjugate pairs $a \pm bi$.

Example: Write a cubic with zeros 2 and $3i$.
Conjugate: $-3i$ is also a zero
Multiply: $(x-2)(x^2+9)$

Answer: $x^3 - 2x^2 + 9x - 18$

5 RATIONAL ROOT THEOREM

Possible rational zeros: $\pm\frac{\text{factors of constant}}{\text{factors of leading coeff.}}$. Test, then divide.

Example: $2x^3 - 3x^2 - 11x + 6 = 0$
Test $x = 3$**:** $54 - 27 - 33 + 6 = 0$
Divide: $2x^2 + 3x - 2 = (2x-1)(x+2)$

Answer: $x = 3,\ \frac{1}{2},\ -2$

6 APPLICATION: OPEN BOX

Cut x-in. squares from the corners of a 12 × 10 in. sheet: $V = x(12-2x)(10-2x)$, $0 < x < 5$.

Example: When is $V = 96$ in.³?
Simplify: $x^3 - 11x^2 + 30x - 24 = 0$
Factor: $(x-2)(x^2 - 9x + 12) = 0$

Answer: $x = 2$ **or** $x = \frac{9-\sqrt{33}}{2} \approx 1.63$