Unit 2 · Linear Functions & Systems Cheat Sheet

Unit 2 focus: slope, forms of a line, parallel and perpendicular lines, and linear models

1 KEY VOCABULARY

slope	Rise over run: $\frac{y_2-y_1}{x_2-x_1}$.	$m=-2$
y-intercept	Value when $x=0$ (the start).	$(0,b)$
residual	Actual − predicted y.	-1.57
correlation r	Strength of a linear fit, $-1\le r\le 1$.	$r\approx 0.99$
feasible region	Points meeting every constraint.	shaded overlap

2 SLOPE & RATE OF CHANGE

Subtract in the same order on top and bottom. In context, slope has units (change in y per unit of x).

Example: Slope through $(-2,5)$ and $(4,-7)$.
Formula: $\frac{-7-5}{4-(-2)}=\frac{-12}{6}$

Answer: $m=-2$

3 THREE FORMS OF A LINE

Form	Equation	Best for
Slope-intercept	$y=mx+b$	graphing, models
Point-slope	$y-y_1=m(x-x_1)$	a point and a slope
Standard	$Ax+By=C$	intercepts; slope $-\frac{A}{B}$

Through $(-2,5)$, $m=-2$: $y-5=-2(x+2)$ → $y=-2x+1$ → $2x+y=1$.

4 PARALLEL & PERPENDICULAR

Parallel: $m_1=m_2$. Perpendicular: flip and change the sign, $m_1m_2=-1$.

Example: Lines through $(6,1)$ related to $y=\frac{2}{3}x-1$.
Parallel: $y=\frac{2}{3}x-3$
Perpendicular: $y=-\frac{3}{2}x+10$

Answer: both pass through $(6,1)$

5 LINEAR MODELS & BEST FIT

Use technology for the regression line; interpret slope and intercept with units. Interpolating is safer than extrapolating.

Example: Weekly miles: $y=2.429x+9.286$, $r\approx 0.986$
Slope: about 2.4 more miles each week
Residual week 3 (15 mi): $15-16.57=-1.57$

Answer: strong positive fit

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Change in y goes on top.
- In point-slope, $x-(-2)$ becomes $x+2$.
- Give units when you interpret slope and intercept.

✘ Common mistakes

- Run over rise: it is rise over run
- Perpendicular: flip only: flip and change the sign
- Trusting extrapolation: far outside the data is unreliable

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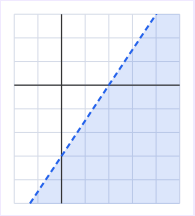
More practice: inequalities, systems, linear programming and three variables

1 LINEAR INEQUALITIES

Solve for y (flip if you divide by a negative). Dashed for $<$, $>$, solid for $\leq$, $\geq$; test a point.

Example: Graph $3x - 2y > 6$.
Solve: $y < \frac{3}{2}x - 3$
Dashed, shade below

Answer: $(0, 0)$ **is not a solution**



2 SYSTEMS: HOW MANY SOLUTIONS?

Lines cross; algebra gives $x = 2$, $y = 3$	**one**
Parallel lines; algebra gives $0 = 4$	**none (inconsistent)**
Same line; algebra gives $0 = 0$	**infinitely many (dependent)**
$3x + 2y = 12$, $5x - 2y = 4$: add → $8x = 16$	$(2, 3)$

3 SYSTEMS IN CONTEXT

Set the two costs equal to find where they match; compare on each side.

Example: Bike A: $C = 15 + 3h$. Bike B: $C = 5 + 5h$.
Equal: $15 + 3h = 5 + 5h$, $h = 5$
Cost: $C = 30$

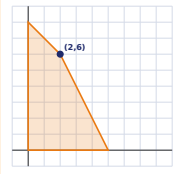
Answer: (5 h, $30); under 5 h B is cheaper

4 SYSTEMS OF INEQUALITIES

Graph each boundary, keep the overlap. Vertices: intersect pairs of boundaries, then check all constraints.

Example: $x + y \leq 8$, $2x + y \leq 10$, $x, y \geq 0$
Vertices: $(0, 0)$, $(5, 0)$, $(2, 6)$, $(0, 8)$

Answer: the shaded quadrilateral



5 LINEAR PROGRAMMING

Vertex	$(0, 0)$	$(5, 0)$	$(2, 6)$	$(0, 8)$
$P = 30x + 20y$	0	150	**180**	160

Test every vertex: the maximum is $P = 180$ at $(2, 6)$. A minimum works the same way.

6 THREE VARIABLES

Eliminate the same variable twice, solve the 2-variable system, back-substitute, check.

Example: $x + y + z = 6$, $2x - y + z = 3$, $x + 2y - z = 2$
(1)+(3), (1)−(2): $2x + 3y = 8$, $-x + 2y = 3$
Solve: $x = 1$, $y = 2$, then $z = 3$

Answer: $(1, 2, 3)$