Unit 4 · Triangle Relationships Cheat Sheet

Unit 4 focus: angle facts, inequalities and special segments in triangles

1 KEY VOCABULARY

median	Vertex to the midpoint of the opposite side.	three in every triangle
altitude	Vertex perpendicular to the opposite side.	$AD \perp BC$
midsegment	Joins the midpoints of two sides.	$DE \parallel BC$
exterior angle	Side and the extension of another side.	outside the triangle

2 ANGLE FACTS

- The three angles add to $180°$.
- Exterior angle = the sum of the two remote interior angles.
- Isosceles: the angles opposite the equal sides are equal.
- Equilateral: every angle is $60°$.

3 EXTERIOR ANGLE THEOREM

An exterior angle equals the sum of the two remote interior angles.

Example: Remote interior angles are $50°$ and $65°$.
Add: $50 + 65$

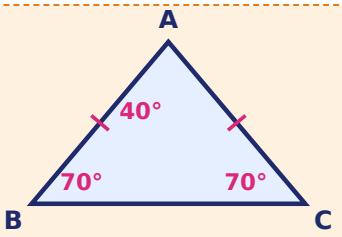
Answer: exterior angle = $115°$

4 ISOSCELES TRIANGLES

Base angles are equal. Subtract the vertex angle from 180, then split the rest.

Example: The vertex angle is $40°$. Find each base angle.
Subtract: $180 - 40 = 140$
Split: $\frac{140}{2}$

Answer: each base angle = $70°$



5 TRIANGLE INEQUALITY

Any two sides must add to more than the third. The third side is between the difference and the sum.

Example: Two sides are 4 and 7. What can the third side be?
Difference: $7 - 4 = 3$
Sum: $4 + 7 = 11$

Answer: $3 < x < 11$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Angles of a triangle add to $180°$.
- Exterior angle uses the two remote angles.
- Check the triangle inequality with the two shorter sides.

✗ Common mistakes

- Adding the adjacent interior angle to the exterior angle: they add to 180, they are not equal
- Using the sum only for the third side: it is also more than the difference
- Calling any median an altitude: a median goes to the midpoint

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More practice: midsegments, medians, bisectors, altitudes and points of concurrency

1 SIDES AND ANGLES

The longest side is opposite the largest angle. The shortest side is opposite the smallest angle.

Example: Angles are 50°, 60°, 70°. Which side is longest?
Largest angle: 70°

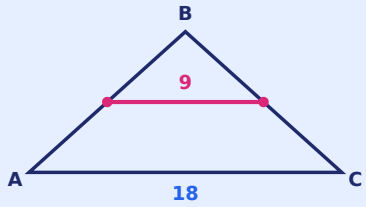
Answer: the side opposite the 70° angle

2 MIDSEGMENT THEOREM

A midsegment is parallel to the third side and half as long.

Example: The third side is 18. How long is the midsegment?
Halve: $\frac{18}{2}$

Answer: midsegment = 9



3 MEDIANS AND THE CENTROID

The medians meet at the centroid. It is $\frac{2}{3}$ of the way from each vertex to the midpoint.

Example: A median is 12 long. How far is the centroid from the vertex?
Multiply: $\frac{2}{3} \cdot 12$

Answer: 8 from the vertex, 4 from the midpoint

4 PERPENDICULAR BISECTOR

Every point on the perpendicular bisector of a segment is the same distance from both endpoints.

Example: $PA = 3x + 2$ and $PB = 5x - 6$ for a point P on the bisector of AB.
Set equal: $3x + 2 = 5x - 6$
Solve: $x = 4$
Substitute: $PA = 3(4) + 2$

Answer: $PA = PB = 14$

5 POINTS OF CONCURRENCY

Point	Lines that meet	Property
centroid	medians	$\frac{2}{3}$ from each vertex
circumcenter	perpendicular bisectors	equal distance to the vertices
incenter	angle bisectors	equal distance to the sides
orthocenter	altitudes	inside, on or outside the triangle

6 CHECK YOURSELF

- Do the angles add to 180°?
- Is the longest side opposite the largest angle?
- Did I use the two remote angles for the exterior angle?
- Which special segment is it: median, altitude or bisector?