Unit 1 · Functions & Transformations Cheat Sheet

Unit 1 focus: function notation, domain and range, parent functions and transformations

1 KEY VOCABULARY

function	Each input has exactly one output.	$\{(1,3),(2,5)\}$
domain / range	All inputs / all outputs.	$[2,\infty)$
parent function	Simplest function of a family.	$y=x^2$
composite	$(f\circ g)(x)=f(g(x))$	g first
inverse	f^{-1} undoes f; reflect over $y=x$.	$(a,b)\to(b,a)$

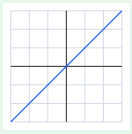
2 NOTATION, DOMAIN & RANGE

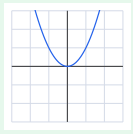
Interval notation: [ ] includes, ( ) excludes, ∞ always gets ( ). Radicand ≥ 0; denominator $\neq 0$.

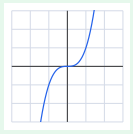
Example: $f(x)=2x^2-3x+1$. Find $f(-2)$.
Substitute: $2(-2)^2-3(-2)+1=8+6+1$
Domain of $\sqrt{x-2}+1$**:** $[2,\infty)$, range $[1,\infty)$

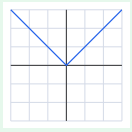
Answer: $f(-2)=15$

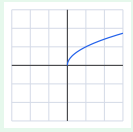
3 PARENT FUNCTIONS

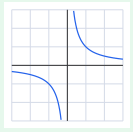

y = x


y = x²


y = x³


y = |x|


y = √x


y = 1/x

Domains: all reals except $\sqrt{x}$ $[0,\infty)$ and $\frac{1}{x}$ $x\neq 0$.

4 TRANSFORMATION RULES

$f(x)+k$	**up** k
$f(x-h)$	**right** h **(**$f(x+3)$**: left 3)**
$-f(x)$ / $f(-x)$	**reflect in x-axis / y-axis**
$a\cdot f(x)$, $\lvert a\rvert>1$	**vertical stretch**
$f(bx)$, $\lvert b\rvert>1$	**horizontal compression by** $\frac{1}{b}$

5 TRANSFORMATION EXAMPLE

Move key points: $(x,y)\to(x-3,\ -2y+1)$.

Example: $g(x)=-2\sqrt{x+3}+1$ from $\sqrt{x}$
Left 3, reflect, stretch by 2, up 1
Points: $(0,0)\to(-3,1)$, $(1,1)\to(-2,-1)$, $(4,2)\to(1,-3)$

Answer: D $[-3,\infty)$**, R** $(-\infty,1]$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Horizontal changes do the opposite: $f(x+3)$ moves left.
- Stretch/reflect before shifting up or down.
- Track key points $(x,y)\to(\text{new } x,\text{new } y)$.

✗ Common mistakes

- $f(x)+3$ vs $f(x+3)$: up 3 vs left 3
- $-f(x)$ vs $f(-x)$: x-axis vs y-axis
- $f(x)$ as $f\cdot x$: it is the output at x

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More practice: operations, composition, piecewise functions and inverses

1 OPERATIONS ON FUNCTIONS

Operation	For $f(x) = 2x + 1$ and $g(x) = x^2 - 3$
$(f + g)(x)$	$x^2 + 2x - 2$
$(f - g)(x)$	$-x^2 + 2x + 4$
$(f \cdot g)(x)$	$2x^3 + x^2 - 6x - 3$
$\left(\frac{f}{g}\right)(x)$	$\frac{2x+1}{x^2-3}$, $x \neq \pm\sqrt{3}$

2 COMPOSITION

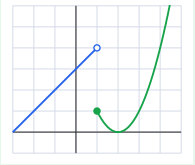
Inside out: do g first, then put its output into f. Order matters.

Example: $f(g(x))$ and $g(f(x))$
$f(g(x))$**:** $2(x^2 - 3) + 1 = 2x^2 - 5$
$g(f(x))$**:** $(2x + 1)^2 - 3 = 4x^2 + 4x - 2$
$(f \circ g)(2)$**:** $f(1) = 3$
Answer: $f \circ g \neq g \circ f$

3 PIECEWISE FUNCTIONS

Use the rule whose condition the input meets. Graph each piece only on its interval (open circle for $<$, closed for $\leq$).

Example: $h(x) = x + 3$ if $x < 1$; $(x - 2)^2$ if $x \geq 1$
$h(0)$**:** $0 + 3 = 3$
$h(4)$**:** $(4 - 2)^2 = 4$
Answer: $h(0) = 3,\ h(4) = 4$



4 FINDING AN INVERSE

Write y, swap x and y, solve for y. Verify: $f(f^{-1}(x)) = x$.

Example: $f(x) = 3x - 5$
Swap: $x = 3y - 5$
Solve: $y = \frac{x + 5}{3}$
Answer: $f^{-1}(x) = \frac{x + 5}{3}$

5 INVERSES & ONE-TO-ONE

- (a, b) on f means (b, a) on f^{-1}; graphs mirror over $y = x$.
- Domain and range swap.
- **Horizontal line test**: if a horizontal line hits twice, there is no inverse function.
- $y = x^2$ needs $x \geq 0$ to have the inverse $\sqrt{x}$. $f^{-1}(x) \neq \frac{1}{f(x)}$.

6 WRITING THE EQUATION OF A GRAPH

Name the parent, read the vertex for h, k, use one more point for a.

Example: Parabola, vertex $(1, 4)$, through $(0, 2)$.
Form: $y = a(x - 1)^2 + 4$
Use $(0, 2)$**:** $2 = a + 4,\ a = -2$
Answer: $y = -2(x - 1)^2 + 4$