Unit 12 · Linear Models & Rates Cheat Sheet

Unit 12 focus: find rates of change and write linear models

1 LINEAR WORDS

Term	Meaning / quick example
Rate of change	Change in y per 1 of x (the slope).
y-intercept	Value when x = 0: a fee or start.
Fixed cost	Same no matter how many.
Variable cost	Grows with each unit.

2 SLOPE FROM TWO POINTS

$m = \frac{y_2 - y_1}{x_2 - x_1}$ with units: dollars per hour.

Example: 2 hours: \$210; 4 hours: \$330.
Rate: $\frac{330 - 210}{4 - 2} = 60$
Fee: $210 - 60(2) = 90$

Answer: \$60 per hour plus a \$90 fee

3 LINEAR MODEL

$y = mx + b$
m = rate (per unit); b = fixed starting amount.

Truck: \$40 plus \$0.80 a mile → $C = 0.80d + 40$.
Budget \$100: $0.80d + 40 = 100$ → $d = 75$ miles.

4 GOALS WITH “AT LEAST”

Write an inequality; round whole units up to reach a goal.

Example: \$120 saved, \$16 an hour, goal at least \$900.
Inequality: $120 + 16h \geq 900$
Solve: $h \geq 48.75$

Answer: 49 hours

5 READING A MODEL

Model	Means
C = 12m + 35	\$35 to join, \$12 per month
B = 150 − 6.5w	\$150 card, −\$6.50 per week
P = 9q − (600 + 3.2q)	profit on q plates

Units tell the story: rate units are “per” something.

6 CHECK A LINEAR MODEL

- Does x = 0 give the fee or starting value?
- Does x = 1 add exactly one rate?
- Are the units in the answer?
- Is rounding up or down sensible for whole items?

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More practice: compare plans, find break-even points and read tiered prices

1 BREAK-EVEN: TWO PLANS

Set the costs equal and solve.

Example: A: $20 + $8 per visit. B: $50 + $5 per visit.
Equal: $20 + 8x = 50 + 5x$
Solve: $3x = 30 \Rightarrow x = 10$

Answer: Same cost at 10 visits; B is cheaper above 10

2 BREAK-EVEN: A BUSINESS

Each sale covers $p - v$ of the fixed cost F.

Example: F = $1,800; v = $3.20; p = $9.
Per sale: $9 - 3.20 = 5.80$
Units: $1{,}800 \div 5.80 \approx 310.3$

Answer: 311 plates (round up)

3 PROFIT

$P = (p - v)q - F$

q = 400, p = $9, v = $3.20, F = $1,800:
$5.80(400) - 1{,}800 = 520$ → **$520 profit**

A negative P is a loss.

4 TIERED ELECTRICITY

Each rate applies only inside its tier.

Example: $12 fee; $0.12 to 500 kWh; $0.18 above; 700 kWh.
Tiers: $500(0.12) + 200(0.18) = 96$
Bill: $12 + 96 = 108$

Answer: $108

5 "OR PART OF" AND CAPS

Round partial units up; stop at any cap.

Example: $4 first hour, $2.50 each extra hour or part, $20 max; 3 h 10 min.
Billed hours: 4
Cost: $4 + 3(2.50) = 11.50$

Answer: $11.50 (under the $20 cap)

6 REMINDERS & MISTAKES

✓ Do remember

- Set models equal to compare plans.
- Bill each tier at its own rate.

✘ Common mistakes

- Lower fee = cheaper plan: only below the break-even.
- F ÷ price for break-even: use F ÷ (price − variable cost).
- All usage at the top tier: only usage above the threshold.
- Round down for goals: round up to reach "at least."