Unit 13 · Exponential Growth & Decay Cheat Sheet

Unit 13 focus: model growth and decay with $y = a \cdot b^t$

1 EXPONENTIAL WORDS

Term	Meaning / quick example
Growth factor	1 + rate: 6% → 1.06.
Decay factor	1 − rate: 15% → 0.85.
Initial value	The amount at t = 0.
Depreciation	Loss of value over time.

2 THE MODEL

$y = a(1 + r)^t$ growth · $y = a(1 - r)^t$ decay
a = start, r = rate per period (decimal), t = periods.

$V = 1{,}100(0.80)^t$: a $1,100 laptop losing 20% a year.

3 GROWTH

Multiply by the factor once per period.

Example: $2,000 at 5% a year for 10 years.
Model: $2{,}000(1.05)^{10}$
Value: $\approx 3{,}257.79$

Answer: $3,257.79

4 DEPRECIATION

Keep 1 − r of the value each year.

Example: $24,000 car, 15% a year, 3 years.
Model: $24{,}000(0.85)^3$
Value: $\approx 14{,}739$

Answer: About $14,739

5 RATE FROM TWO VALUES

factor = new ÷ old; rate = |factor − 1|.

Example: Car: $30,000 → $25,800 in one year.
Factor: $25{,}800 \div 30{,}000 = 0.86$
Rate: $1 - 0.86 = 0.14$

Answer: 14% per year

6 READ A MODEL

- $y = 3{,}200(1.045)^t$ → starts at 3,200 and grows 4.5% per period.
- $y = 650(0.88)^t$ → starts at 650 and decays 12% per period.
- b > 1 grows; 0 < b < 1 decays.

Unit 13 · Exponential Growth & Decay Cheat Sheet

More practice: compare linear and exponential change, doubling and half-life

1 LINEAR OR EXPONENTIAL?

Table	Test	Type
100, 130, 160, 190	differences +30	linear
100, 120, 144, 172.8	ratios × 1.2	exponential
500, 450, 405, 364.5	ratios × 0.9	exponential decay

Same difference → linear. Same ratio → exponential.

2 FLAT RAISE VS. PERCENT RAISE

Percent growth always passes linear growth eventually.

Example: \$40,000: +\$2,500 a year vs. +5% a year.
Year 10, flat: $40{,}000 + 2{,}500(10) = 65{,}000$
Year 10, percent: $40{,}000(1.05)^{10} \approx 65{,}156$

Answer: The percent raise is ahead by year 10

3 RULE OF 72

$$\text{doubling time} \approx \frac{72}{\text{percent rate}}$$

8% → $72 \div 8 = 9$ years. \$1,000 becomes about \$2,000 in 9 years and \$4,000 in 18.

A quick estimate; for the exact whole year, find the first t with $(1 + r)^t \geq 2$.

4 HALF-LIFE

Count half-lives, then halve that many times.

Example: 320 mg, half-life 6 hours, after 18 hours.
Half-lives: $18 \div 6 = 3$
Left: $320\left(\frac{1}{2}\right)^3 = 40$

Answer: 40 mg

5 DOUBLING LADDER

Doublings	Multiplier
1	× 2
2	× 4
3	× 8
5	× 32

A \$1,500 coin that doubles every 8 years is worth $1{,}500(2^3) = 12{,}000$ after 24 years.

6 REMINDERS & MISTAKES

✓ Do remember

- Factor = 1 ± rate, not the rate.
- Count periods carefully.

✗ Common mistakes

- 10% twice = 20%: it is 21% ($1.1^2 = 1.21$).
- Decay reaches \$0: it only gets closer to 0.
- Subtract r% of the original each year: that is linear, not exponential.
- Rule of 72 is exact: it is an estimate.