Unit 8 · Exponential Functions Cheat Sheet

Unit 8 focus: exponential functions, growth and decay

1 KEY VOCABULARY

exponential function	The variable is in the exponent.	$f(x) = 3 \cdot 2^x$
initial value	The starting amount a (value at $x = 0$).	$a = 3$
growth factor	$b > 1$: multiply by b each step.	1.05 = 5% growth
decay factor	$0 < b < 1$: the amount shrinks.	0.8 = 20% decay
asymptote	A line the graph approaches but never reaches.	$y = 0$

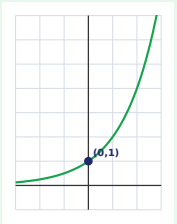
2 THE EXPONENTIAL FORM

Part	Meaning
a	initial value (the y-intercept)
b	the factor y is multiplied by each time x goes up 1
$b = 1 + r$	growth at rate r
$b = 1 - r$	decay at rate r

$y = 200(1.10)^x$: starts at 200 and grows 10% each step.

3 GROWTH VS. DECAY

Growth $b > 1$: rises left to right. **Decay** $0 < b < 1$: falls. Both cross the y-axis at $(0, a)$ and approach the asymptote $y = 0$.



y = 2ˣ



y = (½)ˣ

4 EVALUATE

Exponent first, then multiply by a.

Example: $f(x) = 3(2)^x$. Find $f(4)$.
Substitute: $3(2)^4 = 3(16)$
Answer: 48

5 LINEAR OR EXPONENTIAL?

	Linear	Exponential
pattern	add the same amount	multiply by the same factor
table	x: 0, 1, 2 → y: 5, 8, 11	x: 0, 1, 2 → y: 5, 10, 20
equation	$y = mx + b$	$y = a \cdot b^x$

An increasing exponential eventually passes any linear function.

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- a is the value when $x = 0$.
- Percent → decimal: 6% growth → $b = 1.06$.
- Check for a constant **ratio** (divide consecutive y-values).

✗ Common mistakes

- $3(2)^4 \neq 6^4$: do the power first: $3 \cdot 16$
- 20% decay is not $b = 0.2$: it is $b = 0.8$
- Subtracting y-values: that tests linear, not exponential

Unit 8 · Exponential Functions Cheat Sheet

More practice: writing models, compound interest, geometric sequences and solving equations

1 WRITE FROM A TABLE

Find a (the value at $x = 0$) and b (divide consecutive outputs).

Example: x: 0, 1, 2, 3 → y: 5, 10, 20, 40

a**:** 5

b**:** $10 \div 5 = 2$

Answer: $y = 5(2)^x$

2 GROWTH & DECAY IN CONTEXT

Growth: $y = a(1 + r)^t$. Decay: $y = a(1 - r)^t$.

Example: A town of 8,000 grows 3% per year. Population after 5 years?

Model: $y = 8000(1.03)^t$

$t = 5$**:** $8000(1.03)^5 \approx 9274$

Answer: about 9,274 people

3 COMPOUND INTEREST

$A = P\left(1 + \frac{r}{n}\right)^{nt}$, with n compounding periods per year.

Example: $1,000 at 6% compounded monthly for 2 years.

Substitute: $1000\left(1 + \frac{0.06}{12}\right)^{24}$

Answer: $1,127.16

4 GEOMETRIC SEQUENCES

Multiply by the common ratio r: $a_n = a_1 \cdot r^{n-1}$.

Example: $3, 6, 12, 24, \ldots$: find a_8.

r**:** $6 \div 3 = 2$

$n = 8$**:** $3 \cdot 2^7 = 384$

Answer: 384

5 SOLVING WITH A COMMON BASE

Write both sides with the same base, then set the exponents equal.

Example: Solve $8^x = 2^{x+6}$.

Rewrite: $2^{3x} = 2^{x+6}$

Exponents: $3x = x + 6,\ x = 3$

Answer: $x = 3$

6 KEY FEATURES OF THE GRAPH

- For $y = a \cdot b^x$ with $a > 0$: **domain** all real numbers. **Range**: $y > 0$.
- **y-intercept**: $(0, a)$.
- **Asymptote**: $y = 0$.
- Half-life: the amount halves each period, $b = \frac{1}{2}$.