Unit 6 · Exponential & Log Functions Cheat Sheet

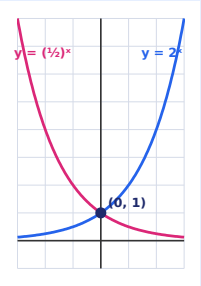
Unit 6 focus: exponential functions, the number e and the logarithm

1 KEY VOCABULARY

exponential function	$f(x) = a \cdot b^x$, $b > 0$, $b \neq 1$.	$3(1.05)^x$
e	Irrational constant ≈ 2.71828; continuous growth.	Pe^{rt}
logarithm	$\log_b x = y$ means $b^y = x$: the log is an exponent.	$\log_2 8 = 3$
common / natural log	$\log x$ is base 10; $\ln x$ is base e.	$\log 100 = 2$
extraneous	A result that fails in the original (log of a number ≤ 0).	$x = -2$

2 GROWTH & DECAY

$f(x) = a \cdot b^x$. Growth $b = 1 + r > 1$; decay $b = 1 - r$. Domain all reals, range $y > 0$ ($a > 0$), y-intercept $(0, a)$, asymptote $y = 0$.



3 TRANSFORMATIONS

$g(x) = a \cdot b^{x-h} + k$: right h, up k (the asymptote moves to $y = k$), $a < 0$ reflects.

Example: $f(x) = 2^{x+1} - 3$
Shift: left 1, down 3
Asymptote: $y = -3$; y-int $2^1 - 3 = -1$

Answer: range $y > -3$

4 THE NUMBER E & INTEREST

n times a year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Continuously: $A = Pe^{rt}$.

Example: \$2,000 at 5% for 6 years
Quarterly: $2000(1.0125)^{24} \approx 2694.70$
Continuous: $2000e^{0.3} \approx 2699.72$

Answer: \$2,694.70 and \$2,699.72

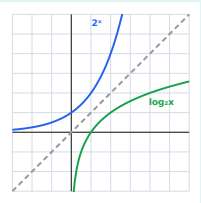
5 LOG ↔ EXPONENTIAL FORM

Exponential	Logarithmic
$2^5 = 32$	$\log_2 32 = 5$
$9^{1/2} = 3$	$\log_9 3 = \frac{1}{2}$
$10^{-2} = 0.01$	$\log 0.01 = -2$
$e^4 = x$	$\ln x = 4$

Special values: $\log_b 1 = 0$, $\log_b b = 1$, $\log_b b^x = x$, $b^{\log_b x} = x$.

6 LOG GRAPHS

$y = \log_b x$ is the inverse of $y = b^x$. Domain $x > 0$, range all reals, asymptote $x = 0$, through $(1, 0)$ and $(b, 1)$. $\log_3(2x - 6)$ needs $x > 3$.



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More practice: log properties, solving equations and modeling

1 PROPERTIES OF LOGS

Rule	Statement
Product	$\log_b(MN) = \log_b M + \log_b N$
Quotient	$\log_b \frac{M}{N} = \log_b M - \log_b N$
Power	$\log_b(M^p) = p \log_b M$
Change of base	$\log_b x = \frac{\ln x}{\ln b}$

✘ $\log(a+b) \neq \log a + \log b$. $\log_5 40 = \frac{\ln 40}{\ln 5} \approx 2.2920$.

2 EXPANDING & CONDENSING

Expand $\log_2 \frac{8x^3}{y}$	$3 + 3\log_2 x - \log_2 y$
Condense $2\ln x + \ln 5 - \ln y$	$\ln \frac{5x^2}{y}$
$\log_6 4 + \log_6 9 = \log_6 36$	2

3 SOLVING EXPONENTIAL EQUATIONS

Same base: set exponents equal. Otherwise isolate the power and take a log of both sides.

Example: $2 \cdot 5^x = 70$
Isolate: $5^x = 35$
Log: $x = \dfrac{\log 35}{\log 5}$
Answer: $x \approx 2.209$

4 SOLVING LOG EQUATIONS (CHECK!)

Condense, convert to exponential form, solve, and reject answers that make a log argument ≤ 0.

Example: $\log_2 x + \log_2(x-2) = 3$
Condense: $x(x-2) = 2^3 = 8$
Solve: $(x-4)(x+2) = 0$
Answer: $x = 4$ **(reject -2)**

5 MODELING FORMULAS

Growth or decay	$y = a(1 \pm r)^t$
Continuous	$A = Pe^{rt}$
Half-life h	$N = N_0\left(\frac{1}{2}\right)^{t/h}$
Doubling time (continuous)	$t = \frac{\ln 2}{r}$
pH	$-\log[H^+]$

6 REAL-WORLD EXAMPLE

Isolate the exponential part, then take $\ln$ (or $\log$) of both sides.

Example: $P = 500e^{0.04t}$. When is $P = 2000$?
Isolate: $e^{0.04t} = 4$
ln: $t = \dfrac{\ln 4}{0.04}$
Answer: about 34.7 years