Unit 5 · Set Theory Cheat Sheet

Unit 5 focus: set equality, the empty set, union, intersection, difference, complements and Venn diagrams

1 OPERATIONS

Name	Notation	Elements
union	$A \cup B$	in A or B (or both)
intersection	$A \cap B$	in both
difference	$A - B$	in A, not in B
complement	A^c or A'	in the universe U, not in A
disjoint	$A \cap B = \varnothing$	no common elements

Always state the universe before taking a complement: in $U = \mathbb{Z}$ the complement of the evens is the odds.

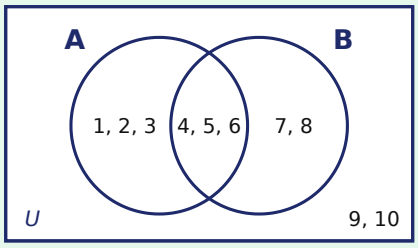
2 WORKED EXAMPLE

$U = \{1, \ldots, 10\}$, $A = \{1, \ldots, 6\}$, $B = \{4, \ldots, 8\}$.

Example: Find each set.
$A \cap B$**:** $\{4, 5, 6\}$
$A - B$**:** $\{1, 2, 3\}$
$(A \cup B)'$**:** $\{9, 10\}$

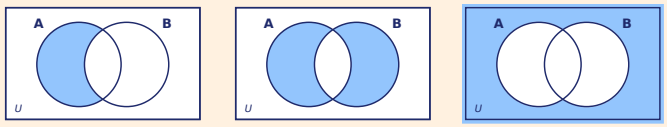
Answer: $A' \cap B' = \{9, 10\}$ **too**

3 VENN DIAGRAM REGIONS



The diagram shows the example: each element goes in exactly one region. $|A \cup B| = |A| + |B| - |A \cap B| = 6 + 5 - 3 = 8$.

4 SHADING PRACTICE



$A - B$ $\quad A \oplus B$ $\quad (A \cup B)'$

Three sets: $A \cap (B \cup C)$ shades the regions of A that also lie in B or C.

5 INCLUSION-EXCLUSION

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Example: How many integers from 1 to 100 are divisible by 3 or 5?
Count: $33 + 20 - 6$
Overlap: multiples of 15: 6

Answer: 47

6 COMMON MISTAKES

✔ Do remember

- List each element once.
- A − B only contains elements of A.
- Endpoints matter for intervals: $[1, 4] - (2, 6) = [1, 2]$.

✘ Common mistakes

- $(A \cap B)' = A' \cap B'$: it is $A' \cup B'$
- Complement with no universe: state U
- Counting the overlap twice: subtract $|A \cap B|$

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More practice: power sets, element-argument proofs, set identities and disproving set statements

1 POWER SETS

$\mathcal{P}(A)$ is the set of all subsets of A; $|\mathcal{P}(A)| = 2^{|A|}$.

Example: $\mathcal{P}(\{a, b\})$
Subsets: $\varnothing, \{a\}, \{b\}, \{a, b\}$
Note: $\{a\} \in \mathcal{P}(A)$ but $a \notin \mathcal{P}(A)$

Answer: 4 elements

2 PROVING A SUBSET

To show $X \subseteq Y$: let $x \in X$ be arbitrary and show $x \in Y$.

Example: Prove $A - B \subseteq A \cap B'$.
Let: $x \in A - B$
Then: $x \in A$ and $x \notin B$
So: $x \in A$ and $x \in B'$

Answer: $x \in A \cap B'$

3 SET IDENTITIES

Law	Identity
commutative	$A \cup B = B \cup A$
distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan	$(A \cup B)' = A' \cap B'$
absorption	$A \cup (A \cap B) = A$
complement	$A \cup A' = U,\ A \cap A' = \varnothing$
set difference	$A - B = A \cap B'$

Prove an identity by showing both inclusions ($\subseteq$ and $\supseteq$) or by a chain of laws.

4 PROVING SET EQUALITY

Chain of laws, naming each one.

Example: Show $A - (A - B) = A \cap B$.
Difference law: $A \cap (A \cap B')'$
De Morgan: $A \cap (A' \cup B)$
Distribute: $(A \cap A') \cup (A \cap B) = \varnothing \cup (A \cap B)$

Answer: $A \cap B$

5 DISPROVING WITH A COUNTEREXAMPLE

False claim	Counterexample
$A - B = B - A$	$A = \{1\},\ B = \varnothing$
$A \cup B = A$	$A = \varnothing,\ B = \{1\}$
$(A \cup B) - B = A$	$A = B = \{1\}$
$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$	$A = \{1\},\ B = \{2\}$: $\{1, 2\}$ is missing

An example where the claim works proves nothing; you need specific sets where it **fails**.

6 CHALLENGE

True or false for all sets: (1) $A \subseteq B \Rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$ **true**: if $X \subseteq A$ then $X \subseteq B$. (2) $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$ **false for every A, B**: $\varnothing$ is on the left but never on the right. (3) How many subsets of $\{1, \ldots, 7\}$ have no two consecutive numbers? $F_9 = 34$.