Unit 10 · Linear Programming Cheat Sheet

Unit 10 focus: writing constraints, graphing systems of inequalities, feasible regions and objective functions

1 VOCABULARY

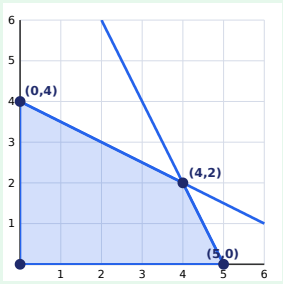
decision variables	the amounts you choose	x tables, y chairs
constraint	an inequality limiting the variables	$2x + y \le 40$
feasible region	points meeting every constraint	the shaded polygon
objective function	the quantity to maximize or minimize	$P = 30x + 50y$
corner point	where boundary lines meet	$(4, 2)$

2 WRITING CONSTRAINTS

Words	Inequality
at most 60 items	$x + y \le 60$
sales of at least \$400 at \$8 and \$5	$8x + 5y \ge 400$
chairs at least 4 times tables	$y \ge 4x$
at most 30% deluxe (x) of all items	$x \le 0.3(x + y)$
cannot make negative amounts	$x \ge 0,\ y \ge 0$

Check a ratio with numbers: 2 tables need at least 8 chairs, so $y \ge 4x$, not $x \ge 4y$.

3 GRAPHING A SYSTEM



$2x + y \le 10$, $x + 2y \le 8$, $x, y \ge 0$. Solid lines for $\le, \ge$; dashed for $<, >$. Test $(0, 0)$ to choose the side.

4 CORNER-POINT METHOD

If a linear objective has an optimum on the region, it occurs at a corner point.

Example: Maximize $P = 3x + 4y$ on the region shown.
Corners: (0, 0), (5, 0), (4, 2), (0, 4)
Values: 0, 15, 20, 16

Answer: max $P = 20$ at (4, 2)

5 FINDING A CORNER

Solve the two boundary equations together.

Example: $2x + y = 10$ and $x + 2y = 8$
Eliminate: $4x + 2y = 20$ minus $x + 2y = 8$
So: $3x = 12$, $x = 4$, $y = 2$

Answer: (4, 2)

6 COMMON MISTAKES

✔ Do remember

- Evaluate the objective at every corner.
- Check that every candidate corner is feasible.
- Use profit = price − cost per unit.

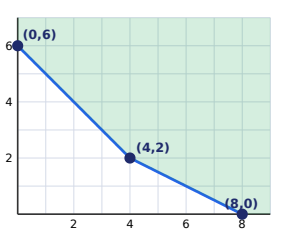
✘ Common mistakes

- "At most" as $\ge$: use $\le$
- Picking the corner with the biggest coordinates: compare objective values
- Testing only one constraint: a solution meets all of them

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More practice: corner points, maximizing and minimizing, special cases and interpreting solutions

1 MINIMIZING ON AN UNBOUNDED REGION



Minimize $C = 2x + 3y$ with $x + y \geq 6$, $x + 2y \geq 8$: C = 18, 14, 16, so **min 14 at (4, 2)**. There is **no maximum**: the region is unbounded.

2 SPECIAL CASES

Case	Sign	Example
unique optimum	one corner is best	max $3x + 4y$ above
multiple optima	two adjacent corners tie: the whole edge is optimal	max $x + 2y$ on $x + 2y \leq 12,\ 3x + 2y \leq 18$: (0, 6) and (3, 4.5) both give 12
unbounded	region extends in a direction that improves the objective	max $x + y$ with $x + y \geq 4$
infeasible	no point meets every constraint	$x + y \leq 3$ and $x + y \geq 5$

3 A COMPLETE PROBLEM

Chairs (x): 2 h cutting, 1 h finishing, \$30 profit. Tables (y): 1 h cutting, 3 h finishing, \$50 profit. 40 h cutting, 45 h finishing.

Example: Maximize $P = 30x + 50y$ s.t. $2x + y \leq 40$, $x + 3y \leq 45$
Corners: (0, 0), (20, 0), (15, 10), (0, 15)
Values: 0, 600, 950, 750

Answer: 15 chairs and 10 tables: \$950

4 INTERPRETING THE SOLUTION

Idea	Meaning
optimal point	how much of each item to make
optimal value	the best profit or cost, with units
binding constraint	used exactly at the optimum (both constraints above)
slack	unused amount: at (20, 0), finishing uses 20 of 45 h: slack 25
whole numbers	if the optimum is fractional, check nearby feasible integer points

5 LEVEL LINES

Set $P = k$: $3x + 4y = k$ is a line with slope $-\frac{3}{4}$. Increasing k slides it away from the origin. The last corner it touches is the maximum. If the level lines are parallel to an edge, every point on that edge is optimal.

6 CHALLENGE

For $P = ax + y$ $(a > 0)$ on the triangle with corners $(0,0), (4,0), (0,6)$: values $0, 4a, 6$. The maximum is at $(4,0)$ when $a > \frac{3}{2}$, at $(0,6)$ when $a < \frac{3}{2}$, and along the whole edge when $a = \frac{3}{2}$.