# Unit 9 · Election Theory Cheat Sheet

**Unit 9 focus:** preference schedules and four named voting methods, fairness criteria

## 1 PREFERENCE SCHEDULE

| Voters | 14 | 10 | 8 | 3 |
|---|---|---|---|---|
| 1st | A | C | B | C |
| 2nd | B | B | C | A |
| 3rd | C | A | A | B |

35 voters, so a **majority** is 18 (more than half). First-place votes: A 14, C 13, B 8 — no majority.

## 2 FOUR METHODS, FOUR WINNERS?

| Method (rule) | Result for the schedule |
|---|---|
| **Plurality**: most first-place votes | A (14) |
| **Borda**: 3, 2, 1 points | A 66, **B 75**, C 69 |
| **Plurality with elimination**: drop the fewest first places | drop B (8 → C): **C** 21–14 |
| **Pairwise comparisons**: 1 point per head-to-head win | B beats A 18–17 and C 22–13: **B** |

Name the method **and** the tie-breaking rule before counting.

## 3 FAIRNESS CRITERIA

| Criterion | Says | Can fail under |
|---|---|---|
| majority | a majority candidate wins | Borda |
| Condorcet | a candidate who wins every head-to-head wins | plurality, Borda, elimination |
| monotonicity | gaining support never hurts a winner | plurality with elimination |
| independence of irrelevant alternatives | removing a loser does not change the winner | all four methods |

**Arrow's theorem:** with 3 or more candidates no ranked method satisfies every fairness condition.

## 4 CONDORCET CHECK

Compare every pair head-to-head.

**Example:** Schedule above
**A vs B:** 17–18 (B)
**A vs C:** 14–21 (C)
**B vs C:** 22–13 (B)

**Answer: B is the Condorcet winner; plurality (A) and elimination (C) violate the Condorcet criterion**

## 5 WEIGHTED VOTING

| Term | In [6: 4, 3, 2] |
|---|---|
| quota $q$: more than half the total, at most the total | total 9: valid quotas 5 to 9 |
| dictator: weight ≥ quota | none |
| veto power: others total < quota | P1 (3 + 2 = 5 < 6) |
| dummy: never critical | none |
| winning coalitions | {P1, P2}, {P1, P3}, {P1, P2, P3} |

## 6 COMMON MISTAKES

**✔ Do remember**

- Majority means more than half.
- Borda uses every position on every ballot.
- Keep eliminating until someone has a majority.

**✘ Common mistakes**

- Plurality = majority: a plurality can be under half
- Largest weight = dictator: needs weight ≥ quota
- Power ∝ weight: count critical appearances

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**More practice:** weighted voting, coalitions and power, fair division and apportionment

## 1 BANZHAF POWER INDEX

Count how often each player is **critical** (its removal turns a winning coalition into a losing one), then divide by the total.

**Example:** [6: 4, 3, 2]
**Critical:** P1 in all three coalitions; P2 in {P1, P2}; P3 in {P1, P3}
**Counts:** 3, 1, 1 (total 5)

**Answer:** $\frac{3}{5}, \frac{1}{5}, \frac{1}{5}$

## 2 FAIR DIVISION

| Method | How it works |
|---|---|
| divider-chooser (2) | one cuts two pieces equal to them; the other chooses |
| lone divider (N) | divider cuts N pieces; choosers bid; standoffs recombined |
| sealed bids (items) | highest bid wins; cash settles fair shares; surplus split |
| markers (items in a row) | markers split the row into N equal-value segments |

A **fair share** is worth at least $\frac{1}{N}$ of the total **in that player's own opinion**.

## 3 SEALED BIDS EXAMPLE

Two heirs; house and car.

**Example:** Ann bids house $300, car $120; Ben bids house $240, car $180.
**Items:** Ann gets the house, Ben the car
**Fair shares:** Ann $210, Ben $210
**Settle:** Ann owes $90; Ben is owed $30; surplus $60 split

**Answer: Ann pays $60 to Ben**

## 4 APPORTIONMENT METHODS

| Method | Rule |
|---|---|
| standard divisor | $SD = \frac{\text{total population}}{\text{seats}}$; quota = population ÷ SD |
| Hamilton | round quotas down, then give leftover seats by largest remainder |
| Jefferson | modified divisor; round **down** |
| Webster | modified divisor; round to **nearest** |
| Adams | modified divisor; round **up** |

## 5 HAMILTON EXAMPLE

Towns 450, 330, 220; 10 seats.

**Example:** $SD = 100$: quotas 4.5, 3.3, 2.2
**Lower quotas:** 4, 3, 2 = 9 seats
**Leftover 1 seat:** largest remainder 0.5

**Answer: 5, 3, 2**

## 6 PARADOXES AND CHALLENGE

**Alabama paradox**: more seats, a state loses one. Hamilton with populations 130, 690, 410: 13 seats → 2, 7, 4; 14 seats → **1**, 8, 5. **Population** and **new-states** paradoxes also affect Hamilton; divisor methods avoid them but can violate the quota rule (Balinski–Young: no method does everything).