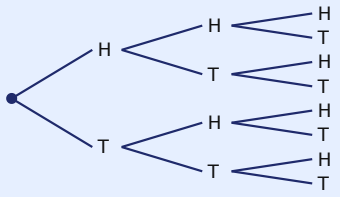
Unit 7 · Counting and Probability Cheat Sheet

Unit 7 focus: sample spaces, events, lists and arrays, possibility trees, addition and multiplication rules

1 SAMPLE SPACES AND EVENTS

sample space S	all outcomes	two coins: HH, HT, TH, TT
event E	a subset of S	"at least one H": 3 outcomes
equally likely	every outcome has the same chance	a fair die
mutually exclusive	events with no common outcome	roll 1 / roll 6

2 POSSIBILITY TREES



Three coin tosses give $2 \cdot 2 \cdot 2 = 8$ leaves. Trees also handle processes that stop early, such as a best-of-three match (6 outcomes: AA, ABA, ABB, BAA, BAB, BB).

3 LISTS: THE FENCEPOST RULE

The list $m, m+1, \ldots, n$ has $n - m + 1$ integers.

Example: How many multiples of 7 are between 100 and 500?
Multiples: $7 \cdot 15 = 105$ to $7 \cdot 71 = 497$
Count: $71 - 15 + 1$

Answer: 57

4 ADDITION AND MULTIPLICATION

Rule	Use when	Example
multiplication	a sequence of steps ("and")	3 shirts and 4 pants: 12 outfits
addition	disjoint cases ("or")	7 mysteries or 5 biographies: 12
inclusion-exclusion	overlapping cases	$\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$
complement	"at least one"	$2^8 - 1 = 255$ bit strings with a 1

5 STATE THE ASSUMPTIONS

Order?	Repetition?	Count	Example (n = 10, r = 3)
yes	yes	n^r	lock codes: 1000
yes	no	$P(n,r) = \frac{n!}{(n-r)!}$	gold, silver, bronze: 720
no	no	$C(n,r) = \frac{n!}{r!\,(n-r)!}$	committees: 120
no	yes	$C(n+r-1, r)$	3 scoops of 10 flavors: 220

6 COMMON MISTAKES

✓ Do remember

- Decide order, repetition and replacement first.
- Handle restricted positions first (no leading 0).
- Use the complement for "at least one".

✘ Common mistakes

- Adding for "and": multiply
- $40 - 15$ integers in 15...40: $40 - 15 + 1 = 26$
- Counting a team as ordered: divide by $r!$

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More practice: permutations and combinations, equally likely probability and counting with probability

1 PERMUTATIONS AND COMBINATIONS

Repeated identical objects: divide by the factorial of each repeat count.

Example: Arrangements of MISSISSIPPI
Letters: 11 total; I 4, S 4, P 2
Count: $\frac{11!}{4!\,4!\,2!}$

Answer: 34650

2 EQUALLY LIKELY PROBABILITY

$P(E) = \frac{|E|}{|S|}$ when outcomes are equally likely; $0 \le P(E) \le 1$ and $P(E') = 1 - P(E)$.

Example: Two fair dice: $P(\text{sum} = 8)$
Outcomes: (2,6), (3,5), (4,4), (5,3), (6,2)
Divide: $\frac{5}{36}$

Answer: $\frac{5}{36}$

3 TWO DICE SUMS

sum	2	3	4	5	6	7	8	9	10	11	12
ways	1	2	3	4	5	6	5	4	3	2	1

The 11 sums are **not** equally likely; the 36 ordered pairs are. $P(7) = \frac{6}{36} = \frac{1}{6}$.

4 WITH VS WITHOUT REPLACEMENT

Use the same counting model for favorable and total outcomes.

Example: Bag: 3 red, 2 blue. Draw 2. P(both red)?
With replacement: $\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$
Without: $\frac{C(3,2)}{C(5,2)} = \frac{3}{10}$

Answer: $\frac{9}{25}$ **vs** $\frac{3}{10}$

5 COUNTING + PROBABILITY

Question	Answer
5 coin tosses: P(exactly 3 heads)	$\frac{C(5,3)}{2^5} = \frac{10}{32} = \frac{5}{16}$
Committee of 4 from 6 boys, 4 girls: P(2 girls)	$\frac{C(4,2)C(6,2)}{C(10,4)} = \frac{90}{210} = \frac{3}{7}$
Seat 4 people at random: P(Ann next to Bo)	$\frac{2 \cdot 3!}{4!} = \frac{1}{2}$

6 CHALLENGE: BIRTHDAYS

Assume 365 equally likely birthdays, independent, no leap day. For n people, $P(\text{all different}) = \frac{365 \cdot 364 \cdots (366-n)}{365^n}$.
With 23 people, $P(\text{some match}) \approx 0.507$ — more than half!