# Unit 2 · The Logic of Compound Statements Cheat Sheet

**Unit 2 focus:** connectives, truth tables, logical equivalence, conditionals and their negations

## 1 CONNECTIVES

| Symbol | Read | True when |
|---|---|---|
| $\neg p$ | not $p$ | $p$ is false |
| $p \wedge q$ | $p$ and $q$ | both true |
| $p \vee q$ | $p$ or $q$ (inclusive) | at least one true |
| $p \oplus q$ | exclusive or | exactly one true |
| $p \to q$ | if $p$ then $q$ | not ($p$ T and $q$ F) |
| $p \leftrightarrow q$ | $p$ if and only if $q$ | same truth value |

Order of operations: $\neg$ first, then $\wedge$, then $\vee$, then $\to$, then $\leftrightarrow$. Use parentheses when unsure.

## 2 TRUTH TABLE ROWS

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $p \to q$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|
| **T** | T | T | T | T | T |
| **T** | F | F | T | **F** | F |
| **F** | T | F | T | T | F |
| **F** | F | F | F | T | T |

Standard order: TT, TF, FT, FF. With $n$ variables a table has $2^n$ rows.

## 3 KEY EQUIVALENCES

| Law | Equivalence |
|---|---|
| De Morgan | $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$ |
| Conditional | $p \to q \equiv \neg p \vee q$ |
| Contrapositive | $p \to q \equiv \neg q \to \neg p$ |
| Distributive | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| Absorption | $p \vee (p \wedge q) \equiv p$ |
| Biconditional | $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ |

## 4 SHOWING EQUIVALENCE

Two forms are equivalent when their truth columns match in **every** row.

**Example:** Is $\neg(p \wedge q)$ equivalent to $\neg p \wedge \neg q$?
**Columns:** FTTT vs FFFT
**Row TF:** first is T, second is F

**Answer: Not equivalent**

## 5 NEGATING CONDITIONALS

$\neg(p \to q) \equiv p \wedge \neg q$. The negation of an if-then statement is an **and** statement, not another if-then.

**Example:** Negate: “If it is sunny, then we go to the beach.”
**Pattern:** $p \wedge \neg q$
**Words:** It is sunny and we do not go to the beach.

**Answer: sunny and no beach**

## 6 TAUTOLOGY OR CONTRADICTION

| Form | Type |
|---|---|
| $p \vee \neg p$ | tautology (always T) |
| $p \wedge \neg p$ | contradiction (always F) |
| $(p \wedge (p \to q)) \to q$ | tautology |
| $p \to q$ | neither (contingent) |

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**More practice:** converse, inverse and contrapositive; valid arguments, rules of inference and fallacies

## 1 CONVERSE, INVERSE, CONTRAPOSITIVE

| Form | Symbols | Equivalent to $p \to q$? |
|---|---|---|
| **statement** | $p \to q$ | — |
| **converse** | $q \to p$ | no |
| **inverse** | $\neg p \to \neg q$ | no (it matches the converse) |
| **contrapositive** | $\neg q \to \neg p$ | **yes** |

Example $p \to q$: "If $x = 2$, then $x^2 = 4$" (true). Converse "If $x^2 = 4$, then $x = 2$" is false ($x = -2$).

## 2 VALID ARGUMENT FORMS

| Rule | Form |
|---|---|
| modus ponens | $p \to q,\ p\ \therefore\ q$ |
| modus tollens | $p \to q,\ \neg q\ \therefore\ \neg p$ |
| transitivity | $p \to q,\ q \to r\ \therefore\ p \to r$ |
| elimination | $p \vee q,\ \neg p\ \therefore\ q$ |
| division into cases | $p \vee q,\ p \to r,\ q \to r\ \therefore\ r$ |
| contradiction rule | $\neg p \to c\ \therefore\ p$ ($c$ always false) |

## 3 TESTING VALIDITY

An argument is **valid** if every row that makes all premises true also makes the conclusion true. One bad row makes it invalid.

**Example:** $p \to q,\ q\ \therefore\ p$
**Find a bad row:** make $p$ false, $q$ true
**Check:** $p \to q$ T, $q$ T, conclusion $p$ F

**Answer: Invalid (converse error)**

## 4 FALLACIES

| Name | Invalid form and example |
|---|---|
| converse error | $p \to q,\ q\ \therefore\ p$: "The grass is wet, so it rained." |
| inverse error | $p \to q,\ \neg p\ \therefore\ \neg q$: "It did not rain, so the grass is dry." |
| inclusive or | $p \vee q,\ p\ \therefore\ \neg q$: both could be true |

An invalid argument can still have a true conclusion; validity is about the **form**.

## 5 COMMON MISTAKES

**✓ Do remember**

- Check every row before calling forms equivalent.
- Write the negation of $p \to q$ as $p \wedge \neg q$.
- Count rows: $2^{(\text{number of variables})}$.

**✗ Common mistakes**

- Treating "or" as exclusive: $\vee$ is inclusive
- Calling the converse equivalent: only the contrapositive is
- De Morgan without switching $\wedge \leftrightarrow \vee$: switch the connective too

## 6 CHALLENGE: KNIGHTS AND KNAVES

Knights always tell the truth; knaves always lie. A says "B is a knave." B says "A and I are the same type."
**Case A knight:** B is a knave, and B's claim ("same type") is false — consistent.
**Case A knave:** B is a knight, but then B's claim is false — impossible.
**Answer:** A is a knight, B is a knave (division into cases plus the contradiction rule).