Unit 9 · Conic Sections Cheat Sheet

Unit 9 focus: circles, parabolas, ellipses and hyperbolas in standard form

1 KEY VOCABULARY

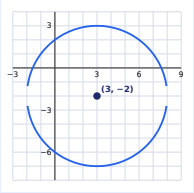
conic section	A curve made by slicing a double cone with a plane.	circle, ellipse
focus	Fixed point(s) used to define the conic.	$(0, 2)$
directrix	Fixed line for a parabola: every point is as far from it as from the focus.	$y = -2$
vertices	Endpoints of the major axis (ellipse) or transverse axis (hyperbola).	$(\pm 5, 0)$
asymptotes	Lines a hyperbola approaches.	$y = \pm\frac{3}{4}x$

2 CIRCLES

$(x-h)^2 + (y-k)^2 = r^2$: center (h, k), radius r. Complete the square in x and y.

Example: $x^2 + y^2 - 6x + 4y - 12 = 0$
Complete: $(x-3)^2 + (y+2)^2 = 12 + 9 + 4$

Answer: center $(3, -2)$**,** $r = 5$

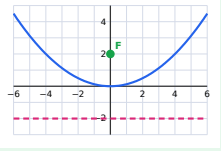


3 PARABOLAS

$(x-h)^2 = 4p(y-k)$: vertex (h, k), focus $(h, k+p)$, directrix $y = k - p$. Opens up if $p > 0$. $(y-k)^2 = 4p(x-h)$ opens sideways.

Example: $y = \frac{1}{8}x^2$
Rewrite: $x^2 = 8y$, so $4p = 8$, $p = 2$

Answer: focus $(0, 2)$**, directrix** $y = -2$

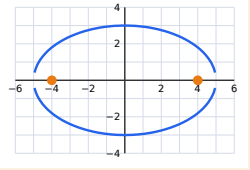


4 ELLIPSES

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, $a > b$: horizontal major axis. Foci: $c^2 = a^2 - b^2$.

Example: $\frac{x^2}{25} + \frac{y^2}{9} = 1$
Axes: vertices $(\pm 5, 0)$, co-vertices $(0, \pm 3)$
Foci: $c^2 = 25 - 9 = 16$

Answer: foci $(\pm 4, 0)$

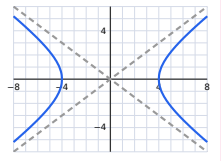


5 HYPERBOLAS

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ opens left–right. Foci: $c^2 = a^2 + b^2$. Asymptotes $y = \pm\frac{b}{a}x$.

Example: $\frac{x^2}{16} - \frac{y^2}{9} = 1$
Vertices: $(\pm 4, 0)$; $c^2 = 25$, foci $(\pm 5, 0)$

Answer: asymptotes $y = \pm\frac{3}{4}x$



6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Ellipse: a^2 is the larger denominator; hyperbola: a^2 is under the positive term.
- Ellipse $c^2 = a^2 - b^2$; hyperbola $c^2 = a^2 + b^2$.
- Add the same amounts to both sides when completing the square.

✗ Common mistakes

- $(x+3)^2 + y^2 = 16$: center $(3, 0)$: it is $(-3, 0)$, $r = 4$
- $r = 16$ for $x^2 + y^2 = 16$: $r = 4$
- $c = a + b$: use squares: $c^2 = a^2 \pm b^2$

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More practice: identifying conics, converting to standard form, focus and directrix, and applications

1 IDENTIFY THE CONIC

$Ax^2 + Cy^2 + Dx + Ey + F = 0$	Conic	Example
$A = C$	circle	$2x^2 + 2y^2 - 8 = 0$
$AC > 0$, $A \neq C$	ellipse	$4x^2 + 9y^2 = 36$
$AC < 0$	hyperbola	$x^2 - 4y^2 = 4$
$A = 0$ **or** $C = 0$	parabola	$y^2 - 8x = 0$

2 CONVERTING TO STANDARD FORM

Group, factor out the squared coefficients, complete each square, divide to get 1.

Example: $4x^2 + 9y^2 - 16x + 18y - 11 = 0$
Complete: $4(x-2)^2 + 9(y+1)^2 = 11 + 16 + 9 = 36$
Divide by 36: center $(2, -1)$, $a = 3$, $b = 2$

Answer: $\dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{4} = 1$

3 FOCUS & DIRECTRIX

The vertex is halfway between the focus and the directrix; p is that distance.

Example: Focus $(2, 3)$, directrix $y = -1$
Vertex: $(2, 1)$, $p = 2$
Check $(6, 3)$**:** 4 from the focus, 4 from the directrix

Answer: $(x-2)^2 = 8(y-1)$

4 VERTICAL ORIENTATION

$\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$	**ellipse, vertices** $(0, \pm 5)$**, foci** $(0, \pm\sqrt{21})$
$\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$	**hyperbola opening up/down, vertices** $(0, \pm 3)$
$y^2 = -12x$	**parabola opening left, focus** $(-3, 0)$

5 GRAPHING CHECKLIST

- **Circle:** center, then r up, down, left, right.
- **Parabola:** vertex, focus, directrix, and the two points $2|p|$ either side of the focus.
- **Ellipse:** center, a along the major axis, b along the minor axis.
- **Hyperbola:** center, vertices, the $2a \times 2b$ box, asymptotes through its corners.

6 APPLICATION: SATELLITE DISH

A parabolic dish reflects signals to its focus. Put the vertex at the origin: $x^2 = 4py$.

Example: A dish is 8 ft wide and 2 ft deep. Where is the receiver?
Point $(4, 2)$**:** $16 = 4p(2)$
Solve: $p = 2$

Answer: 2 ft above the vertex