Unit 2 · Limits and Continuity Cheat Sheet

Unit 2 focus: limit notation, evaluating limits algebraically and graphically

1 KEY VOCABULARY

limit	The value f(x) approaches as x → a.	$\lim_{x\to 2} x^2 = 4$
one-sided limit	Approach from one side.	$\lim_{x\to 0^+}$
indeterminate form	Needs more work.	$\frac{0}{0}$
continuous at a	$\lim_{x\to a} f(x) = f(a)$	polynomials

2 DIRECT SUBSTITUTION

Try plugging in first.

Example: $\lim_{x\to 3}(2x^2 - x)$
Substitute: $18 - 3$
Answer: 15

3 FACTOR AND CANCEL

$\frac{0}{0}$ → factor, cancel, substitute.

Example: $\lim_{x\to 2} \frac{x^2-4}{x-2}$
Factor: $\frac{(x-2)(x+2)}{x-2} = x + 2$
Answer: 4

4 RATIONALIZE

Multiply by the conjugate.

Example: $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$
Conjugate: $\frac{1}{\sqrt{x+9}+3}$
Answer: $\frac{1}{6}$

5 SPECIAL TRIG LIMITS

Limit	Value
$\lim_{x\to 0} \frac{\sin x}{x}$	1
$\lim_{x\to 0} \frac{1-\cos x}{x}$	0
$\lim_{x\to 0} \frac{\sin 5x}{x}$	5

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- $\frac{0}{0}$ means “do more work,” not “no limit.”
- A limit exists only if both one-sided limits agree.
- f(a) need not equal the limit.

✘ Common mistakes

- $\frac{0}{0} = 0$: indeterminate
- $\frac{5}{0}$ → limit is 0: infinite / DNE
- Limit = f(a) always: only if continuous

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More practice: one-sided and infinite limits, continuity and the IVT

1 ONE-SIDED LIMITS

Check each side separately.

Example: $\lim_{x\to 0} \frac{|x|}{x}$
Left: -1
Right: 1
Answer: DNE

2 INFINITE LIMITS

Nonzero ÷ 0 → ±∞; check the sign.

Example: $\lim_{x\to 3^+} \frac{2}{x-3}$
Denominator: small positive
Answer: +∞ (vertical asymptote x = 3)

3 LIMITS AT INFINITY (RATIONAL)

Degrees	Limit
top < bottom	0
top = bottom	ratio of leading coefficients
top > bottom	±∞

$\lim_{x\to\infty} \frac{3x^2+1}{5x^2-x} = \frac{3}{5}$

4 CONTINUITY AT X = A

- 1. f(a) is defined.
- 2. $\lim_{x\to a} f(x)$ exists.
- 3. The limit equals f(a).
- Discontinuities: removable (hole), jump, infinite (asymptote).

5 MAKING A PIECEWISE FUNCTION CONTINUOUS

Set the left and right pieces equal at the break.

Example: f(x) = kx + 1 for x < 2, x^2 for x ≥ 2
Match: 2k + 1 = 4
Answer: $k = \frac{3}{2}$

6 INTERMEDIATE VALUE THEOREM

Continuous on [a, b] → hits every value between f(a) and f(b).

Example: Does $x^3 - x - 1 = 0$ have a root in [1, 2]?
Signs: f(1) = −1, f(2) = 5
Answer: Yes, by the IVT