Unit 6 · The Fundamental Theorem of Calculus Cheat Sheet

Unit 6 focus: FTC part 1 and part 2

1 THE FUNDAMENTAL THEOREM

Part	Statement
Part 1	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
Part 2	$\int_a^b f(x)\,dx = F(b) - F(a)$

2 EVALUATING WITH FTC 2

Antiderivative, then top minus bottom.

Example: $\int_1^3 (2x+1)\,dx$
Antiderivative: $x^2 + x$
Evaluate: (9 + 3) − (1 + 1)

Answer: 10

3 TRIG INTEGRAL

Use exact values.

Example: $\int_0^{\pi/2} \cos x\,dx$
sin x: $\sin\frac{\pi}{2} - \sin 0$

Answer: 1

4 FTC 1 WITH A CHAIN

Plug in the upper limit and multiply by its derivative.

Example: $\frac{d}{dx}\int_2^{x^3} \sqrt{t}\,dt$
Apply: $\sqrt{x^3} \cdot 3x^2$

Answer: $3x^{7/2}$

5 DEFINITE U-SUBSTITUTION

Change the limits to u-values.

Example: $\int_0^1 2xe^{x^2}\,dx$
u = x²: $\int_0^1 e^u\,du$

Answer: e − 1

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- FTC 2: F(b) − F(a), top minus bottom.
- FTC 1: the derivative undoes the integral.
- Change limits when substituting.

✗ Common mistakes

- F(a) − F(b): F(b) − F(a)
- Forgetting the chain factor: multiply by the inner derivative
- Keeping x-limits after u-sub: convert them

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More practice: average value, accumulation functions and net change

1 AVERAGE VALUE

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Example: $f(x) = x^2$ on [0, 3]
Integral: 9
Divide by 3: 3
Answer: 3

2 ACCUMULATION FUNCTION

$g(x) = \int_a^x f(t)\,dt$; g′ = f.

Example: $g(x) = \int_0^x (t^2 - 4)\,dt$: where is g increasing?
g′(x) = x² − 4 > 0: |x| > 2
Answer: x < −2 or x > 2

3 NET CHANGE

$F(b) = F(a) + \int_a^b F\prime(t)\,dt$

Example: Water: 50 gal at t = 0, rate r(t) = 4t gal/min. Amount at t = 3?
Integrate: $\int_0^3 4t\,dt = 18$
Answer: 68 gal

4 DISPLACEMENT VS. DISTANCE

Quantity	Integral
Displacement	$\int_a^b v(t)\,dt$
Total distance	$\int_a^b \lvert v(t)\rvert\,dt$

v(t) = t − 2 on [0, 4]: displacement 0, distance 4.

5 READING G FROM THE GRAPH OF F

f	g
f > 0	g increasing
f changes + to −	g has a max
f increasing	g concave up

6 CHECK YOURSELF

- Did I include the initial amount for net change?
- Distance uses |v|, displacement does not.
- Did I divide by (b − a) for average value?
- Units: rate × time = amount.