Unit 8 · Differential Equations Cheat Sheet

Unit 8 focus: slope fields and verifying solutions

1 KEY VOCABULARY

differential equation	Relates a function and its derivatives.	$\frac{dy}{dx} = 2y$
general solution	Contains + C.	$y = Ce^{2x}$
particular solution	C found from an initial condition.	$y = 3e^{2x}$
slope field	Small slope segments at grid points.	shows solution curves

2 VERIFYING A SOLUTION

Differentiate and substitute.

Example: Is $y = e^{3x}$ a solution of y′ = 3y?
y′: $3e^{3x} = 3y$ ✓
Answer: Yes

3 SLOPE AT A POINT

Plug the point into dy/dx.

Example: $\frac{dy}{dx} = x - y$ at (3, 1)
Compute: 3 − 1
Answer: slope 2

4 READING SLOPE FIELDS

Slopes depend only on x	**same along vertical columns**
Slopes depend only on y	**same along horizontal rows**
dy/dx = 0	**horizontal segments (equilibrium if y = c)**

5 EULER'S METHOD

$y_{new} = y + \frac{dy}{dx}\Delta x$

Example: $\frac{dy}{dx} = x + y$, y(0) = 1, Δx = 0.1. Estimate y(0.1).
Step: 1 + (0 + 1)(0.1)
Answer: 1.1

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Slope fields show slope, not y-values.
- Particular solution: solve for C.
- Check the solution by differentiating.

✗ Common mistakes

- Forgetting + C: add C right after integrating
- Plugging the point in before integrating: integrate first
- Confusing x and y in the slope: use (x, y) in order

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More practice: separation of variables and exponential growth and decay

1 SEPARATION OF VARIABLES

- 1. Get all y's with dy, all x's with dx.
- 2. Integrate both sides (+ C on one side).
- 3. Use the initial condition to find C.
- 4. Solve for y if possible.

2 SEPARATING VARIABLES

Divide by y, multiply by dx.

Example: $\frac{dy}{dx} = xy$, y(0) = 2
Separate: $\int \frac{dy}{y} = \int x\,dx$
Integrate: $\ln|y| = \frac{x^2}{2} + C$
Answer: $y = 2e^{x^2/2}$

3 ANOTHER SEPARABLE EQUATION

Separate, integrate, solve.

Example: $\frac{dy}{dx} = \frac{x}{y}$, y(0) = 3
Integrate: $\frac{y^2}{2} = \frac{x^2}{2} + C$
C: $\frac{9}{2}$
Answer: $y = \sqrt{x^2 + 9}$

4 EXPONENTIAL GROWTH AND DECAY

Model	Solution
$\frac{dy}{dt} = ky$	$y = y_0 e^{kt}$
k > 0	growth
k < 0	decay

5 HALF-LIFE

Use the half-life to find k: $\frac{1}{2} = e^{kT}$.

Example: Half-life 10 years; 80 g at t = 0. Amount at t = 30?
Three half-lives: $80(\frac{1}{2})^3$
Answer: 10 g

6 CHECK YOURSELF

- Did I separate before integrating?
- Did I add + C and solve for it?
- Is my k positive for growth, negative for decay?
- Does my solution pass through the initial point?