Unit 3 · Derivatives Cheat Sheet

Unit 3 focus: the definition of the derivative and basic rules

1 LIMIT DEFINITION

$f\prime(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Example: $f(x) = x^2$
Expand: $\frac{2xh+h^2}{h} = 2x + h$
Answer: $f\prime(x) = 2x$

2 BASIC RULES

Function	Derivative
c	0
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$

3 TRIG DERIVATIVES

Function	Derivative
sin x	cos x
cos x	−sin x
tan x	$\sec^2 x$
sec x	sec x tan x
csc x	−csc x cot x
cot x	$-\csc^2 x$

4 POWER RULE

Rewrite roots and fractions as powers first.

Example: $y = 4x^3 - \frac{2}{x} + \sqrt{x}$
Rewrite: $4x^3 - 2x^{-1} + x^{1/2}$
Answer: $12x^2 + \frac{2}{x^2} + \frac{1}{2\sqrt{x}}$

5 TANGENT LINE

$y - f(a) = f\prime(a)(x - a)$

Example: $f(x) = x^3$ at x = 2
f(2), f′(2): 8 and 12
Answer: y − 8 = 12(x − 2)

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Derivative = slope of the tangent = instantaneous rate.
- Rewrite before differentiating.
- Differentiable ⇒ continuous (not the reverse).

✘ Common mistakes

- $(fg)\prime = f\prime g\prime$: $f\prime g + fg\prime$
- $\frac{d}{dx}e^{3x} = e^{3x}$: $3e^{3x}$
- Corners are differentiable: not at a corner or cusp

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More practice: product, quotient and chain rules, implicit and higher derivatives

1 PRODUCT AND QUOTIENT RULES

Rule	Formula
Product	$(fg)\prime = f\prime g + fg\prime$
Quotient	$(\frac{f}{g})\prime = \frac{f\prime g - fg\prime}{g^2}$
Chain	$(f(g(x)))\prime = f\prime(g(x))\,g\prime(x)$

2 PRODUCT RULE

First′ · second + first · second′.

Example: $y = x^2 \sin x$

Apply: $2x \sin x + x^2 \cos x$

Answer: $2x \sin x + x^2 \cos x$

3 QUOTIENT RULE

“Low d-high minus high d-low, over low squared.”

Example: $y = \frac{x}{x+1}$

Apply: $\frac{(x+1)-x}{(x+1)^2}$

Answer: $\frac{1}{(x+1)^2}$

4 CHAIN RULE

Outside derivative (inside unchanged) × inside derivative.

Example: $y = (3x^2+1)^5$

Apply: $5(3x^2+1)^4 \cdot 6x$

Answer: $30x(3x^2+1)^4$

5 IMPLICIT DIFFERENTIATION

Differentiate both sides; multiply y-terms by $\frac{dy}{dx}$.

Example: $x^2 + y^2 = 25$

Differentiate: $2x + 2y\frac{dy}{dx} = 0$

Answer: $\frac{dy}{dx} = -\frac{x}{y}$

6 INVERSE TRIG DERIVATIVES

Function	Derivative
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$