Unit 7 · Applications of Integrals Cheat Sheet

Unit 7 focus: area between curves

1 AREA BETWEEN CURVES

$\int_a^b (\text{top} - \text{bottom})\,dx$

Example: $y = x$ and $y = x^2$
Intersect: x = 0, 1
Integrate: $\int_0^1 (x - x^2)\,dx$
Answer: $\frac{1}{6}$

2 INTEGRATING IN Y

$\int_c^d (\text{right} - \text{left})\,dy$

Example: $x = y^2$ and x = 4
Limits: y = −2 to 2
Integrate: $\int_{-2}^2 (4 - y^2)\,dy$
Answer: $\frac{32}{3}$

3 SETTING UP AREA

- Sketch the region.
- Find the intersection points (limits).
- Decide dx (top − bottom) or dy (right − left).
- If the curves cross, split the integral.

4 CURVES THAT CROSS

Split at each crossing so every piece is positive.

Example: $\int_0^{2\pi} |\sin x|\,dx$
Split: 2 + 2
Answer: 4

5 CHOOSING DX OR DY

Top and bottom curves are functions of x	**dx**
Left and right curves are functions of y	**dy**
Would need two integrals in x	**try dy**

6 QUICK REMINDERS & COMMON MISTAKES

✔ Do remember

- Area is always positive.
- Use intersection points as limits.
- Top minus bottom, right minus left.

✘ Common mistakes

- Bottom − top: top − bottom
- Using x-limits with dy: y-limits
- Not splitting where curves cross: split the integral

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More practice: volumes by discs, washers and cross sections

1 VOLUME FORMULAS

Method	Volume
Disc	$\pi \int_a^b R(x)^2\,dx$
Washer	$\pi \int_a^b (R^2 - r^2)\,dx$
Cross sections	$\int_a^b A(x)\,dx$

2 DISC METHOD

Radius = distance from the axis to the curve.

Example: Rotate $y = \sqrt{x}$, 0 ≤ x ≤ 4, about the x-axis.
Integrate: $\pi \int_0^4 x\,dx$
Answer: 8π

3 WASHER METHOD

Outer radius squared minus inner radius squared.

Example: Region between y = x and $y = x^2$ about the x-axis.
Integrate: $\pi \int_0^1 (x^2 - x^4)\,dx$
Answer: $\frac{2\pi}{15}$

4 ROTATING ABOUT ANOTHER LINE

Radius = |curve − axis|.

Example: Rotate $y = x^2$, 0 ≤ x ≤ 1, about y = −1.
Radii: R = x² + 1 (outer), r = 1 (inner)
Answer: $\pi \int_0^1 ((x^2+1)^2 - 1)\,dx = \frac{13\pi}{15}$

5 CROSS-SECTION AREAS

Shape (side s)	Area
Square	s^2
Equilateral triangle	$\frac{\sqrt{3}}{4}s^2$
Semicircle (diameter s)	$\frac{\pi}{8}s^2$

Squares on y = √x, 0 ≤ x ≤ 4: V = ∫ x dx = 8.

6 CHECK YOURSELF

- Did I square each radius separately (R² − r²)?
- Is my radius measured from the axis of rotation?
- Did I include π for discs and washers?
- Does my sketch match my setup?