Unit 4 · Applications of Derivatives Cheat Sheet

Unit 4 focus: related rates, motion and linearization

1 RELATED RATES

Write an equation, differentiate with respect to t, THEN substitute.

Example: A circle's radius grows 2 cm/s. How fast is area growing when r = 5?
Differentiate: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
Answer: 20π **cm²/s**

2 MOTION

Quantity	Meaning
s(t)	position
v(t) = s′(t)	velocity
a(t) = v′(t)	acceleration
speed	|v(t)|

Speeding up when v and a have the same sign.

3 LINEARIZATION

$L(x) = f(a) + f\prime(a)(x - a)$

Example: Estimate $\sqrt{4.1}$.
a = 4: $2 + \frac{1}{4}(0.1)$
Answer: 2.025

4 PARTICLE AT REST

Set v(t) = 0.

Example: $s(t) = t^3 - 6t^2 + 9t$
v(t): $3t^2 - 12t + 9 = 3(t - 1)(t - 3)$
Answer: t = 1 and t = 3

5 L'HÔPITAL'S RULE

Form	Do
$\frac{0}{0}$ or $\frac{\infty}{\infty}$	$\lim \frac{f\prime}{g\prime}$
other forms	rewrite first

$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim \frac{e^x}{1} = 1$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Substitute numbers only AFTER differentiating.
- Include units with rates.
- L'Hôpital uses f′/g′, not the quotient rule.

✗ Common mistakes

- Plug in r = 5 before differentiating: differentiate first
- Speed = velocity: |velocity|
- L'Hôpital on 1 ÷ 0: only 0/0 or ∞/∞

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More practice: extrema, the MVT, curve sketching and optimization

1 FINDING EXTREMA

- Critical points: $f'(x) = 0$ or undefined.
- First derivative test: sign change + to − → max, − to + → min.
- Second derivative test: $f'' > 0$ → min, $f'' < 0$ → max.
- Closed interval: compare critical points AND endpoints.

2 ABSOLUTE EXTREMA

Evaluate f at critical points and endpoints.

Example: $f(x) = x^3 - 3x$ on [0, 2]
$f' = 0$: $x = 1$
Values: $f(0) = 0$, $f(1) = -2$, $f(2) = 2$
Answer: max 2, min −2

3 CONCAVITY

$f''(x)$	Graph
> 0	concave up (∪)
< 0	concave down (∩)
changes sign	point of inflection

$f(x) = x^3$: $f'' = 6x$, inflection at $x = 0$.

4 MEAN VALUE THEOREM

Some c in (a, b) has $f\prime(c) = \frac{f(b)-f(a)}{b-a}$.

Example: $f(x) = x^2$ on [1, 3]
Average slope: $\frac{9-1}{2} = 4$
$2c = 4$: $c = 2$
Answer: c = 2

5 OPTIMIZATION

Write a function of ONE variable, differentiate, test.

Example: Maximize area with 40 m of fence (rectangle).
$A = x(20 - x)$: $A' = 20 - 2x = 0$
Answer: 10 m × 10 m, 100 m²

6 CHECK YOURSELF

- Did I check endpoints on a closed interval?
- Did I justify max/min with a test?
- Did I answer the question asked (with units)?
- Did I differentiate before substituting?