Unit 5 · Antiderivatives and Definite Integrals Cheat Sheet

Unit 5 focus: antiderivatives, indefinite integrals and u-substitution

1 BASIC ANTIDERIVATIVES

Integral	Result
$\int x^n\,dx$	$\frac{x^{n+1}}{n+1} + C$ (n ≠ −1)
$\int \frac{1}{x}\,dx$	$\ln\lvert x\rvert + C$
$\int e^x\,dx$	$e^x + C$
$\int \cos x\,dx$	sin x + C
$\int \sin x\,dx$	−cos x + C
$\int \sec^2 x\,dx$	tan x + C

2 POWER RULE FOR INTEGRALS

Add 1 to the exponent, divide by the new exponent. +C!

Example: $\int (6x^2 - 4x + 3)\,dx$

Integrate: $2x^3 - 2x^2 + 3x$

Answer: $2x^3 - 2x^2 + 3x + C$

3 U-SUBSTITUTION

Let u = the inside; replace dx using du.

Example: $\int 2x(x^2+1)^4\,dx$

u = x² + 1: du = 2x dx → $\int u^4\,du$

Answer: $\frac{(x^2+1)^5}{5} + C$

4 INITIAL VALUE PROBLEM

Integrate, then use the point to find C.

Example: $f\prime(x) = 3x^2$, f(1) = 5

Integrate: $f(x) = x^3 + C$

1 + C = 5: C = 4

Answer: $f(x) = x^3 + 4$

5 MORE INTEGRALS

Integral	Result
$\int \frac{1}{1+x^2}\,dx$	$\tan^{-1} x + C$
$\int \frac{1}{\sqrt{1-x^2}}\,dx$	$\sin^{-1} x + C$
$\int e^{kx}\,dx$	$\frac{1}{k}e^{kx} + C$

6 QUICK REMINDERS & COMMON MISTAKES

✓ Do remember

- Always add + C to indefinite integrals.
- Check by differentiating your answer.
- With u-sub, every x must become u.

✗ Common mistakes

- $\int x^{-1}dx = \frac{x^0}{0}$: $\ln\lvert x\rvert$
- $\int \sin x\,dx = \cos x$: −cos x + C
- Leaving x and u mixed: substitute fully

Unit 5 · Antiderivatives and Definite Integrals Cheat Sheet

More practice: Riemann sums, definite integrals and their properties

1 RIEMANN SUMS

Method	Height from
Left	left endpoint
Right	right endpoint
Midpoint	midpoint
Trapezoid	$\frac{\Delta x}{2}(f_0 + 2f_1 + \cdots + f_n)$

Increasing f: left underestimates, right overestimates.

2 RIGHT RIEMANN SUM

$\Delta x = \frac{b-a}{n}$, add f(right) · Δx.

Example: $f(x) = x^2$ on [0, 2], n = 4
Δx = 0.5: 0.5(0.25 + 1 + 2.25 + 4)

Answer: 3.75

3 DEFINITE INTEGRAL AS AREA

Use geometry when the graph is a known shape.

Example: $\int_0^4 (x+1)\,dx$
Trapezoid: $\frac{1}{2}(1+5)(4)$

Answer: 12

4 INTEGRAL PROPERTIES

Property	Rule
Reverse limits	$\int_b^a f = -\int_a^b f$
Same limits	$\int_a^a f = 0$
Split	$\int_a^c f = \int_a^b f + \int_b^c f$
Constant	$\int_a^b kf = k\int_a^b f$

5 USING PROPERTIES

Combine the given pieces.

Example: $\int_0^3 f = 7$, $\int_0^5 f = 2$. Find $\int_3^5 f$.
Split: 2 − 7

Answer: −5

6 CHECK YOURSELF

- Did I add + C?
- Is Δx = (b − a) ÷ n?
- Area below the x-axis counts as negative.
- Did I differentiate to check?